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# Learners' performance in Mathematics: A case study of public high schools, South Africa

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## ABSTRACT

Mathematics is fundamental to national prosperity in providing tools for understanding science, technology, engineering and economics. It is essential in public decision-making and for participation in the knowledge economy. Mathematics equips pupils with uniquely powerful ways to describe, analyse and change the world. It can stimulate moments of pleasure and wonder for all pupils when they solve a problem for the first time, discover a more elegant solution, or notice hidden connections. This study investigated a societal problem – the ongoing poor performance in mathematics. The study described what is going on by means of statistical methods and reported in statistical language and hence, descriptive quantitative research paradigm infused with content analysis (cartoons) was adopted. The sample for the study comprised of a total of ten underperforming (failing) secondary schools obtained by simple random sampling; 50 Grade 12 mathematics educators and 200 Grade 12 mathematics learners were derived through stratified sampling technique.

**Keywords**: mathematics, performance, pass rate, underperforming secondary school

## INTRODUCTION

Mathematics is widely acknowledged as one of the cornerstones of future development and prosperity. Professor Kader Asmal (2003) classifies mathematics as the priority of all priorities; Ibn Khaldun, Muqaddima (1332) as Gouba (2008: slide 2) quotes, stresses that, "Education should be started with mathematics, for it forms well designed brains that are able to reason right"; mathematics is generally accepted as a gateway subject "enabling discipline" (Pandor, 2006a:2); President Thabo Mbeki (2000; 2001), emphasised the centrality of mathematics as part of our human development strategy; as for Justina (1991), mathematics is increasingly recognised as one of the most reliable indicators for measuring socio-economic and geo-political development among nations; and finally, Azikiwe puts it, "Mathematics is the bedrock of science while science is the necessity for technological and industrial development" (Betiku, 1999:49). In sharp contrast to these perceptions, the low mathematics pass rates at Grade 12 level are not only a source of frustrations and embarrassment for the learners concerned, but also reflect a low-level return for substantial investment made by the Government, communities, and parents in the education of their children.

The education system takes a lion's share of resources, placing South Africa at or near the top of the international league in terms of proportion of national resources (GDP) devoted to the education spending. Since the transition to democracy, resources devoted to school education have increased considerably and large resource shifts have taken place to the poorer schools (Van der Berg, 2001), yet outputs of successful matriculants or of those matriculating with university exemption are stagnating or declining (Van der Berg and Burger, 2002). Accordingly, the Minister of Finance, Mr Trevor Manuel (Budget 2008) posits that "Education is our central objective of broadening opportunity and fighting poverty. This budget priorities school building, early childhood education, school books and educator remuneration. The education spending accounts for R105.7 billion." Furthermore, Mr Trevor Manuel (Budget 2009) encapsulates that, "Government's contribution to public education remains our single largest investment, because we know that it is the key to reducing poverty and accelerating long-term economic growth." On the same note, Mr Pravin Gordhan (current Minister of Finance) reiterates that, "Education spending remains our largest item of spending; giving meaning to our commitment that it is our number one priority. The total budget for education ... is R165.1 billion."(Budget 2010) Consequently, with about 5.3 percent of gross domestic product and 20 percent of total state expenditure on education, South Africa has one of the highest rates of public investment in education in the world (Budget 2008; 2009; 2010).

In addition, a number of programmes and initiatives have been put in place: the South African National Department of Education in 2000, introduced a National Strategy for Mathematics, Science and Technology Education (now called DINALEDI project); the North West Department of Education established Mathematics, Science and Technology Unit (MSTU); the Western Cape Education Department, launched the Khanya Project. The President, Jacob Zuma (2009) reiterates the non-negotiables, "Teachers should be in school, in class, on time, teaching, with no neglect of duty and abuse of pupils! The children should be in class, on time, learning, be respectful of their teachers and each other, and do their homework." On the other hand, the Minister of Basic Education, Mrs Angelina Matsie Motshekga, MP, in 2010, launched the Action Plan to 2014: Towards the Realisation of Schooling 2025, in order to avoid ad hoc and fragmented interventions. (Report on the National Senior Certificate Examination Results: 2010).

Furthermore commitment in education are witnessed in a number of initiatives and programmes such as HIV/AIDS awareness programmes in schools, reducing average class sizes in schools serving lower income communities, adult basic education and training (ABET), increasing school expenditure on school buildings, recapitalising technical high schools, strengthening teacher training programmes, the Fee-free Schools, National Schools Nutrition Programme, Scholar Transport, Saturday classes, winter school, Senior Secondary Intervention Programme (SSIP), Study Mates, Teach South Africa, University bridging courses, learner ships and e-learning channels.

Despite the fact that South Africa spends a large budgetary commitment (about 20% of its annual budget on education), programmes and initiatives, the entire schooling system is characterised with series of low matriculation pass rates especially in gateway subjects (mathematics, science and accounting) (Naidoo, 2004; Coetzee, 2008). The quality of education remains very poor, and the output rate has not improved. The Star (January 8 2010) stresses that the standard of public education is not reflecting a corresponding correlation with the massive investment. The NSC examination of 2008 was the first based on the New Curriculum Statement (Curriculum 2005). The following table (Table1) gives an insight into the national pass rates of all the learning areas (mathematics statistics interpolated as it was the area of study).

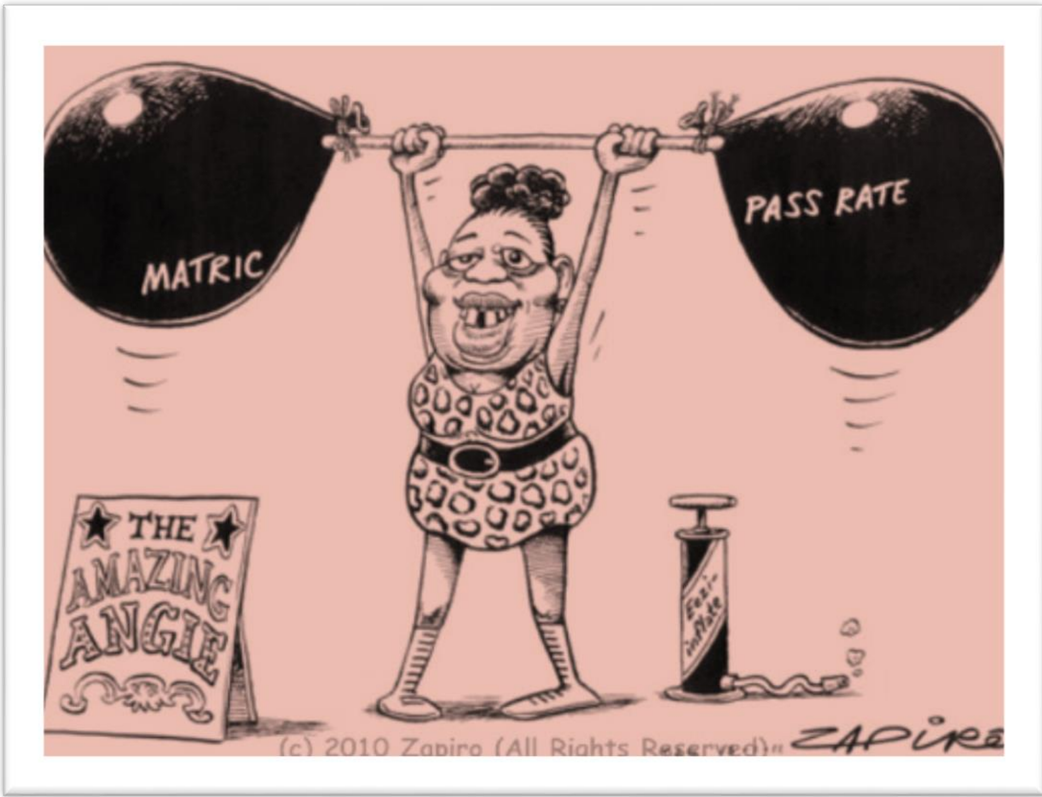
**Table 1**. National Senior Certificate Pass Rates In All Learning Areas (Mathematics statistics Interpolated): 2008 to 2010 Educational Measurement, Assessment and Public Examinations (2010)

| Year | | Total Wrote | Total Passed | % Passed |
|---|---|---|---|---|
| **2008** | All subjects | 554 664 | 345 001 | 62.6 |
| | **Maths** | **298 821** | **136 503** | **45.6** |
| **2009** | All subjects | 580 527 | 351 829 | 60.6 |
| | **Maths** | **290 407** | **133 505** | **46** |
| **2010** | All subjects | 537 543 | 364 513 | 67.8 |
| | **Maths** | **263 034** | **124 749** | **47.4** |

Table 1 above depicts, the first NSC examination achieving a national pass rate of 62.6 percent and national mathematics pass rate of 45.6 percent. Commenting on the Matric Results of 2008, Pandor (2009) best describes matric pass rates as "pathetic" (educationweb.co.za). In 2009, the national pass rate plummeted to 60.6 percent while the national mathematics pass rate increased by 0.4 percent to 46.0 percent. Govender (2010) points out that, "The 60.6 percent pass rate recorded for 2009 is a far cry from the 70.7 percent achieved in 2004. The national matric pass rate of 60.6 percent which has been declining since 2004 is a national disgrace." Motshekga MP, Minister of Basic Education (2010) on announcing South Africa's 2009 National Senior Certificate Results, regrettably admits "This achievement is depressing", and adds that, "matric results are economic failure."

2010 witnessed a national pass rate of 67.8 percent, a massive improvement of 7.2 percent from 2009. More meaningfully, Zapiro's (2010) Cartoon A depicts a proud, Angie Motshekga, Minister of Basic Education, announcing that 67.8% of students passed the 2010 exams – a nearly 7 percent plus increase from the previous year and a "remarkable achievement!" she says.

**Figure 1**. Cartoon A: Showing "THE AMAZING ANGIE: MATRIC PASS RATES"




Shapiro, 2011.Published in Mail & Guardian on 13 Jan

Contrarily to "The amazing Angie: Matric pass rates" an open mind questions, "Was the matric pass rate of 2011 a true reflection of learner achievement?" The obvious answer as according to Cartoon A becomes NO! The balloons and the pump suggest 2011 matric pass rate was inflated. On the same token (Table1), according to Pandor (2011), "The number of learners sitting for Grade 12 mathematics declined from 298 821 in 2008 to 290 407 in 2009 and 263 034 in 2010."

Conclusively, Dr Valley (Wits Policy Units, 2008) criticises the 30% and 40% benchmarks for passes feeling these were too low. He points out that, "We are setting our sights too low. There is nothing to celebrate. Our schooling system is failing our young people and we need to revive it." (http//www.educationweb.co.za)

## METHODOLOGY

This study followed descriptive quantitative survey research and content analysis methodologies. Descriptive quantitative survey research was seen ideal, because it is concerned with the present, although it often considers past events and influences as they relate to current conditions (Cohen and Manion 1994:67); concerned with the relationship of one set of facts to another (Bell 2005:13); mathematical procedures are followed to aggregate and summarise the evidence (Romberg, 1992: 52); a systematic process in which numerical data are utilised to obtain information about the world (Patton, 1990:20). As for Dooley (1990) and Neill (2004), standardised measurement procedures are used to assign numbers to observations, and statistical procedures are used to analyse quantitative data (Durrheim 1999b:96). Furthermore, content analysis or textual analysis was adopted as it uses a sample of images rather than people (Haralambos, Holborn and Heald, 1995:854-855), a technique for making inferences by objectively and systematically identifying specified characteristics of antecedents of communication (Holti, 1969), and texts are studied as to authorship, authenticity or meaning (Neuendorf, 2002). In addition, the results of content analysis allow researchers to identify, as well as quantify, specific ideas, concepts, and their associated patterns, and trends of ideas that occur within a specific group or over time (Krippendorff, 1980 and Weber, 1990). More importantly, the results of content analysis are numbers and percentages and further uncover causes and promote awareness.

## PARTICIPANTS

In this study, the population was 138 Grade 12 mathematics teachers and 1153 Grade 12 mathematics learners at 23 underperforming (failing) secondary schools in Gauteng East district. The sampling of participants of this study began with simple random sampling of 10 underperforming secondary schools. Stratified random sampling was applied to obtain two hundred learners (136 boys and 64 girls) and fifty teachers (17 females and 33 males). The sample responded to a respondent-centred questionnaire with closed and open questions or statements (Burton and Bartlett, 2009:175; Struwig and Stead, 2004:92; De Vos, 1998:89 and Terre Blanche et al, 2006:486). As for content analysis, the corpus (the body of information) was daily and weekly newspapers. The sample of the study was images on the cartoon page - educational cartoons (Weber, 1990; and McNamara, 1998).

## QUANTITATIVE DATA ANALYSIS

According to Neuman (1997:294), a researcher provides tables, graphs and charts to give the reader a condensed picture of the data. The data was coded before computing it. Coding data, according to Neuman (1997:295), means "systematically reorganising data that is computer readable." The respondents' responses were assessed on the basis of their agreement or disagreement with the attitudinal declarative statements, such as; yes or no and Likert's four-point scale, that is; Strongly Agree (SA); Agree (A); Disagree (D); Strongly Disagree (SD) - which forces a decision, were used in the study.

Statistical Programme for Social Sciences (SPSS), one of the most widely used programs for statistical analysis (a) in social sciences and (b) data management (Nie, Bent and Hull; 1970) was used to analyse responses of the learner and the educator questionnaires. The descriptive statistical analysis used in this study consisted of the number of respondents, lists, frequency distribution tables, percentage tables, cross tabulation and statistical graphs of item variables.

## FINDINGS

As the purpose of the present investigation was to explore factors that cause low pass rates in mathematics at Grade 12 level in public high schools; all participants were (mathematics) learners and educators. Five factors (Educational policies - Learning areas; Promotion policy and Educator's and Learners' behaviour - School or classroom discipline; Drugs and alcohol abuse; and Learner pregnancy) resulting from questionnaires' responses and analysis are presented here.

### Findings with regard to the first problem question and the aim of this study: To what extent will educational policies contribute to Grade 12 learners' mathematics pass rates?

**Table 2.** Educational policies

| | A | D | % | % |
|---|---|---|---|---|
| The number of learning areas is too many | 216 | 31 | 88 | 12 |
| Condoned mathematics learners | 174 | 76 | 69 | 31 |

## LEARNING AREAS

To confirm responses from the questionnaires administered about some educational policies in place, Table 2 revealed that the low pass rates in mathematics at Grade 12 level were largely due to the existing educational policies (learning areas condoned learners). To recap the overall responses, out of 250 respondents, 216 respondents (88%) indicated that low pass rates in mathematics at Grade 12 level were greatly due to the number of learning areas prescribed by NCS (a learner must do at least seven subjects at FET). On the other hand, 31 participants (12%) disagreed of which, 21 participants (08%) disagreed and 10 participants (04%) strongly disagreed.

This suggests that the majority of participants, as reflected on the table suggest that mathematics Grade 12 learners were performing poorly due to the number of learning areas. One of the purposes of higher education as stipulated in the White Paper (DoE, 1997:7) is "to contribute to the socialisation of enlightened, responsible and constructively critical citizens" this may suggest subject specialisation - strong discipline (content) base. Accordingly, Muller (2009) sees curriculum in terms of the differences in discipline as well as coherence. He concurs with Biglan's (1973:204) division of academic areas into (a) a single paradigm (making it hard or soft), (b) practical application (in terms of pure or applied), and, finally, whether it falls under life systems or not. In accordance with the thinking, curriculum designers may bring to fore, the introduction of subject specialisation as from Grade 10 and perhaps, a minimum of three learning areas would enhance good pass rates not in mathematics only, but even in other gateway subjects.

## PROMOTION POLICIES

In regards to Promotion Policy (Table 2 above), an overwhelmingly 174 respondents (69%) agreed or strongly agreed with the assertion that “Condoned learners” contribute to low pass rates in mathematics at Grade 12 level in public high schools in Gauteng East District. The 69% was made up of, fifty-eight respondents (23%) who strongly agreed and a further one hundred and sixteen respondents (46%) who agreed. Contrarily, 76 respondents (31%) disagreed; 46 respondents (18%) disagreed and 21 respondents (13%) strongly disagreed.

Indiscriminate promotion from lower classes in schools significantly leads to poor matric pass rates particularly in mathematics. Condonation, according to National Protocol for Assessment Grades 1-12, is the relaxation of promotion requirements as contemplated in paragraph 29(1) (b) of the policy document, pertaining to the programme and promotion requirements of the National Curriculum statement. Condoning (assisting learners to reach matric) to a greater extent, contributes to low matric pass rates in mathematics (as mostly learners are condoned in mathematics from Grade 1up to Grade 12 level).

On the other hand, keeping learners in a grade where they are not doing well is an indication that the education system is not good. A recent survey suggests that 10 percent of the learners across all grades are 3 or more years outside the age-group norm. The Department of Basic Education’s age-group norms state that a child should be seven years in grade 1, eight in grade 2 and so on. Grade repetition has resulted in a large number of over-age learners in our education system instead of high pass rates Meny-Gilbert (the teacher 2010).

According to Ncana (The Times 2010), schools are marked with a significant age differential or age mixing between learners and older adolescents. Meny-Gilbert (2010) talks about multi-age classrooms, and consequently, many teachers struggle to cope with reality. The curriculum for each grade is aimed at a particular pedagogic development stage-premised on children being of a certain age.

Consequently, grade repetition (learners repeating failed grade at most three times) contributes to a significantly large number of over-age learners with spill over effects of many learner pregnancies. These findings confirmed to a certain extent to the findings of Vogel (2003:115) who synthesises it all, “the current system of giving a child a condoned pass is a double-edged sword.”

**Findings regarding the second problem question and the aim of this study: How will the behaviour of educators’ and learners’ affect the overall Grade 12 mathematics pass rates?**

**Table 3:** Behaviour of educators’ and learners’

| | **A** | **D** | **%** | **%** |
|---|---|---|---|---|
| Indiscipline amongst educators’ and learners’ | 153 | 97 | 61 | 39 |

## SCHOOL (CLASSROOM) DISCIPLINE

As evident in Table 3, 60 participants (24%) strongly agreed that “Indiscipline amongst educators’ and learners’” was a contributing factor to low pass rates in mathematics at Grade 12 level in public high schools in Gauteng East District. In addition, 93 participants (37%) agreed, hence 61% of the respondents were in agreement with the assertion. On the other hand, 97 participants (39%) disagreed, of which, 59 participants (24%) disagreed and 38 participants (15%) strongly disagreed. South African society has undergone major social, economic and political changes over the past few years as we have sought to establish a democratic and humane nation. Among the changes in the education sector has been the banning of corporal punishment in all schools under the Convention on the Rights of the Child (CRC) by being a signatory and the African Charter on the Rights and Welfare of the Child (ACRWC) which compels that a child who is subjected to school or parental discipline shall be treated with humanity and with respect for the inherent dignity of the child.

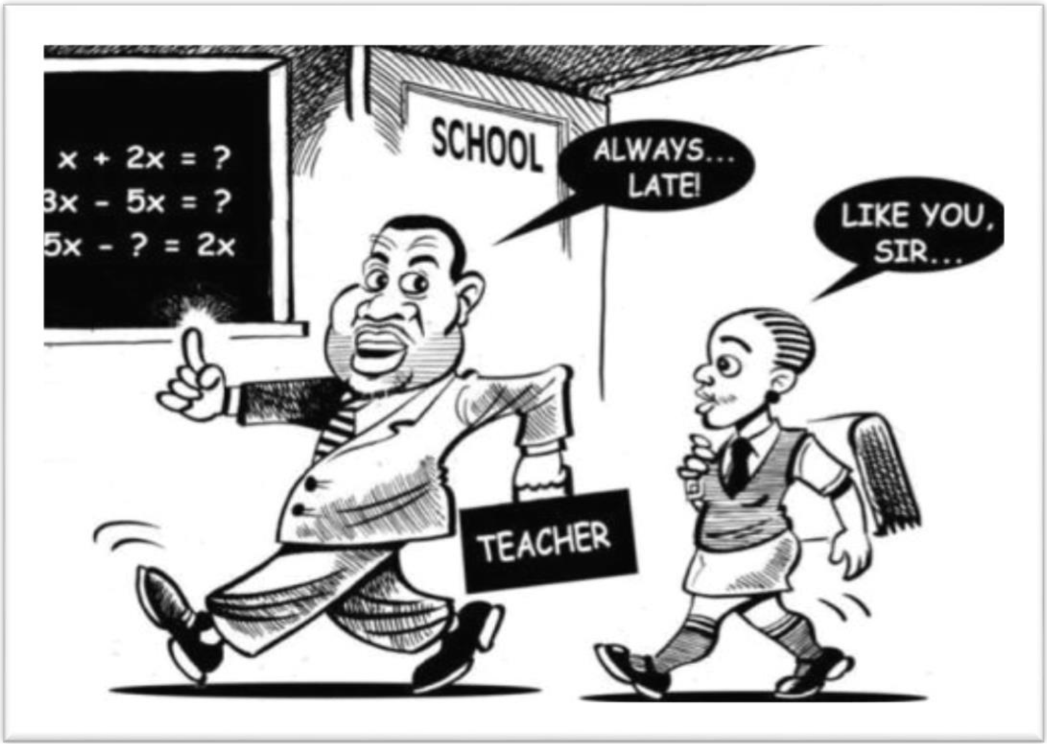
This table (Table 3 above) put it in clear terms how the respondents felt about school discipline. This may suggest that participants had the following in mind when they were completing the questionnaire:

- Section 12 of the South African Constitution states that: Everyone has the right not to be treated or punished in a cruel, inhuman or degrading way.
- The National Education Policy Act (1996**)** says, No person shall administer corporal punishment or subject a student to psychological or physical abuse at any educational institution.

- The South African Schools Act (1996**)** says: (1) No person may administer corporal punishment at a school to a learner; (2) Any person who contravenes subsection 1 is guilty of an offense, and liable on conviction to a sentence which could be imposed for assault.

Findings from the present study were thus, further consistent with Siwela's cartoon. Siwela (Feb 17 2012: The Citizen) rightly captures morning order at a school with the following cartoon. The "teacher", a role model to be, is always... late and subsequently, learners are also coming late to school.

**Figure 2**. Cartoon B: "School: Always... late! Like you' Sir"




Siwela**:** Feb 17 2012: The Citizen

It is worth noting here that lack of discipline in many schools stems from the 1970's where pupils were given the power by revolutionary forces to make the country ungovernable. Pupils to this day, still think they can run the school. In the 1980's there was a policy "Pass one, pass all" and in those circumstances, it was almost impossible to maintain standards. Historically, this crisis has been intensified by the widespread political unrests which in turn eroded discipline in schools. This political unrest was expressed by parents, learners, political organisations and educators struggle against the previous Bantu education under apartheid. De Villiers (1997:76) emphasises this statement when he states that the political factors, especially the role the school played in apartheid played a major role in undermining discipline in the South African Black schools.

The findings of this study, to a very great extent, in South African schools suggests that lack of discipline and self-discipline among educators and high school pupils has probably led to a continuation of unsuccessful learning and teaching and furthermore, perpetuation of low pass rates in mathematics.

**Table 4**. Behaviour of educators' and learners' drug and alcohol abuse

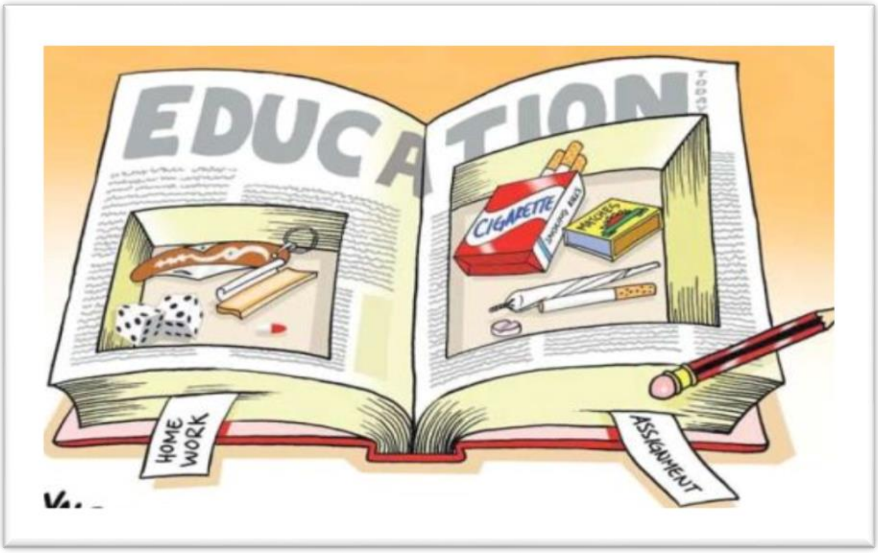
| | A | D | % | % |
|---|---|---|---|---|
| Drugs and alcohol abuse | 154 | 96 | 62 | 38 |

As evident in Table 4, negative behaviour (drugs and alcohol abuse) of educators' and learners' towards teaching and learning was statistically significant in contributing to low pass rates in mathematics at Grade 12 level. The results showed more than half of the respondents (62% of the respondents) agreed that negative behaviour of both educators and learners was one of the main factors causing low pass rates in mathematics. Less than half of the respondents (38%) disagreed with the assertion.

The findings are in line with the results of the most informative study concerning the prevalence of drinking behaviour of youth and adolescents. The results of the study (Rocha-Silva, de Miranda and Erasmus, 1996) revealed that 42.5% of the respondents reported having used alcohol at some stage during their lives. From a study of a sample of 7340 students in Grades 8 to 12 from 16 high schools (Flisher et al, 1993a, b), of the total sample, 53.2% of the respondents reported ever using alcohol. According to a report from the Bureau of Justice (2011), 85% of the teenagers claim that they know where to obtain marijuana, while 29% state that someone has offered or sold them an illegal substance at school.

Nationally, Yalo (2011), pictorially identifies (in the Sowetan newspaper) negative influences on education of many school goers of these days. Learners' drinking attract much attention in the press and hence, Yalo rightly shows the Class of 2011 concentrating on drugs, gambling, smoking, glue sniffing and violence instead of doing homework and school based assessments. The following cartoon (Cartoon C) depicts the education of today.

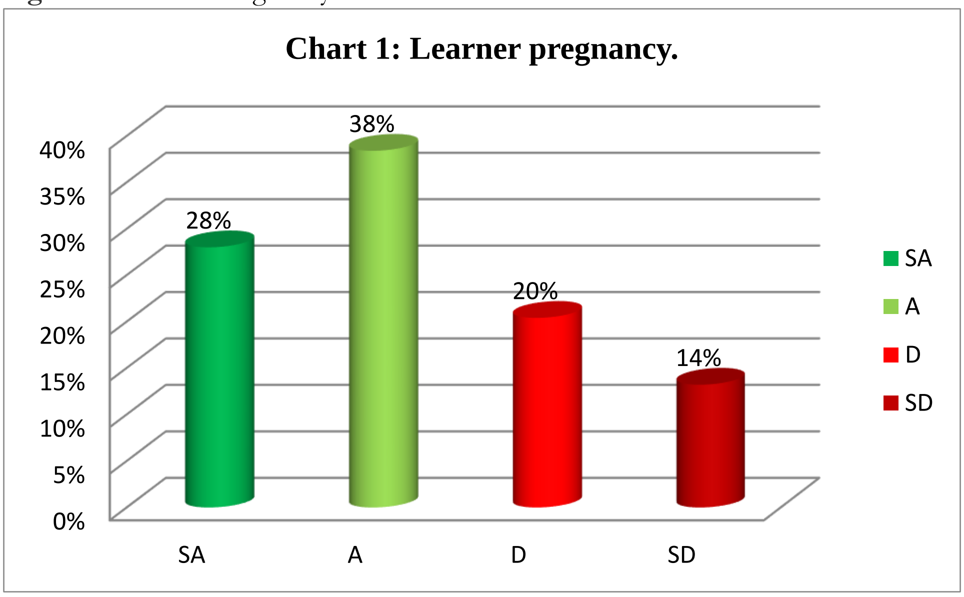
**Figure 3**. Cartoon C: Education today




Yalo | Mar 25, 2011 | Sowetan

Lastly, the findings are in line with the United Nations World Drug Report (2009). The recent United Nations World Drug Report has named South Africa as one of the worlds' drug capitals. Experts have expressed concern that drug abuse is epidemic in South African' schools were the drug level of drug users has dropped from teens to between 9 to 10 (Mohlala, theTeacher 2010). In addition, the consumption of alcoholic beverages has a very long history in South Africa dating back to very ancient times (Gumede, 1995).

**Figure 3.** Learner Pregnancy



As with the results in Chart 1, seventy respondents (28%) strongly agreed that "Learner pregnancy" was a contributory factor to low pass rates in mathematics at Grade 12 level in public high schools in Gauteng East District. In addition, ninety-six respondents (38%) agreed to the assertion. On the other hand, eighty-four respondents (33%) disagreed of which, fifty-one respondents (20%) disagreed and thirty three respondents (14%) strongly disagreed. Teenage pregnancy has emerged as one of the many challenges facing schools. The high agreement to the assertion could have been informed by the following consequences:

- Health (high risk of infant mortality);
- Educational (difficult to learn during pregnancy, absenteeism);
- Economic (exacerbate poverty)
- Social (stigma and discrimination, de-motivated parents, distracting other learners [being envied]).

Petje (2000), in the introductory remarks of Circular 53/2000 highlights teenage pregnancy statistics in South Africa, for instance, 100 000 legal abortions have been carried out since the passing of the choice on Termination of Pregnancy Act 92 of 1996.

Nationally, a similar pattern holds, as Yalo's cartoon shows. Yalo (Feb 22, 2011), reacts to the release of statistics depicting a frightening high instance of schoolgirl pregnancies with the following cartoon in one of the national newspaper - Sowetan.

**Figure 4.** Cartoon D: School pregnancies (Yalo Feb 22, 2011 Sowetan)

The findings are further supported by statuary instruments. According to the South African Constitution, 1996 (Act 108 of 1996) and South African Schools Act, Act number 84 of 1996, a pregnant learner may absent herself from school and be allowed to continue with her school after the delivery of her baby. In terms of this Act (SASA), a pregnant learner may not be expelled from school on the basis of her pregnancy, nor may she be refused admission to school on the basis that she is or was pregnant. Motshekga Angie (Minister of Basic Education) in Ncana (The Times Thursday May 20 2010) concurs with the findings by pointing out that, 'Pregnancies in schools are still through the roof.'

In a rights-based society, young girls who fall pregnant should not be denied access to education and this is entrenched in law in South Africa through the Constitution and Schools Act of 1996. In 2007, the Department of Education released Measures for the Prevention and Management of Learner Pregnancy. Not without controversy, the guidelines continue to advocate for the right of pregnant girls to remain in school, but suggests up to a two year waiting period before girls can return to school in the interest of the rights of the child. Any proposed shift in policy and practice needs to be informed by a well-rounded understanding of the context of teenage pregnancy.

The incidence of teen pregnancy is growing, according to the Human Science Research Council (HSRC), a government think tank. Out of every 1000 girls in school in 2004, 51 were pregnant. The number jumped to 62 in 2008, the most recent data available. Teenage pregnancy has militated against the educational success of girls in South Africa. Statistics show that 4 out of 10 girls become pregnant overall at least once before 20.

However, while it is acknowledged that the population of pregnant and former pregnant learners in public high schools is on the increase due to these positive policy measures (Pandor, 2007), the capacity of schools to cope with this current challenge needs to be established.

## RECOMMENDATIONS

Based on the findings reached in the study, the following recommendations were made.

- No teacher education policy framework: Implement train the educator programme (Human resource development training programmes);
- Approaches to managing absenteeism should be devised in a holistic way, to take account of the broader problems that contribute to absenteeism;
- Department to pronounce clear guidelines about classroom management and discipline strategies;
- A national policy on teenage pregnancy with clear guidelines that safeguard rights to education should be crafted.
- To strengthen the subject specialisation knowledge of the learners at FET, narrowing subjects to at least three as from Grade 10.

## CONCLUSION

The research led to the following conclusions:
The Outcomes-based Education has taught learners that they can pass with minimum mathematics knowledge. OBE develops short memory but does nothing to develop long term knowledge and skills. For learners to competitively participate in the technologically advancing global village, subject specialisation is a prerequisite. According to the findings of present study, learners perform significantly poorly largely due to too many learning areas on offer; indiscipline (both educators and learners); promotion policies; drug and alcohol abuse and learner pregnancy.

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# Investigating Children's Multiplicative Thinking: Implications for Teaching

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## ABSTRACT

Multiplicative thinking is a 'big idea' of mathematics that underpins much of the mathematics learned beyond the early primary school years. This article reports on a recent study that utilised an interview tool and a written quiz to gather data about children's multiplicative thinking. Our research has so far revealed that many primary aged children have a procedural view of multiplicative thinking which we believe inhibits their progress. There are two aspects to this article. First, we present some aspects of the interview tool and written quiz, along with some of findings, and we consider the implications of those findings. Second, we present a key teaching idea and an associated task that has been developed from our research. The main purpose of the article is to promote the development of conceptual understanding of the multiplicative situation as opposed to the teaching of procedures. In doing so, we encourage the explicit teaching of the many connections within the multiplicative situation and between it and other 'big ideas' such as proportional reasoning and algebraic thinking.

**Keywords:** multiplicative thinking, arrays, factors, multiples, interviews

This article has been developed from two papers written and presented by the authors, and published in the proceedings of the 52nd annual conference of the Mathematical Association of Victoria (MAV), Back to the Future, Melbourne, December 3-4, 2015.

## BACKGROUND

The importance of multiplicative thinking as a 'big idea' of mathematics has been well documented (Siemon, Bleckly, & Neal, 2012; Siemon, Breed, Dole, Izard, & Virgona, 2006), as has the importance of 'big ideas' in highlighting the myriad connections within and between them (Charles, 2005; Clarke, Clarke, & Sullivan, 2012). Charles (2005) asserted that 'big ideas' "link numerous mathematical understandings into a coherent whole", and make connections, and that "good teaching should make those connections explicit" (p. 10). Multiplicative thinking is one such 'big idea'.

Multiplicative thinking is vitally important in the development of significant mathematical concepts and understandings such as algebraic reasoning (Brown & Quinn, 2006), place value, proportional reasoning, rates and ratios, measurement, and statistical sampling (Mulligan & Watson, 1998; Siemon, Izard, Breed & Virgona, 2006). Further, Siegler et al. (2012) advocate that knowledge of division and of fractions (another part of mathematics very much reliant on multiplicative thinking) are unique predictors of later mathematical achievement.

### The Difficulties inherent in Multiplicative Thinking

Unfortunately, research (Clark & Kamii, 1996; Siemon, Breed, Dole, Izard, & Virgona, 2006) has found that the label of being 'solid' multiplicative thinkers cannot be applied to most students. Clarke and Kamii (1996) found that 52% of fifth grade students were not 'solid' multiplicative thinkers, and Siemon, Breed, Dole, Izard, and Virgona (2006) established that up to 40% of Year 7 and 8 students performed below curriculum expectations in multiplicative thinking, with at least 25% well below the expected level.

Whereas most students enter school with informal knowledge that supports both counting and early additive thinking (Sophian & Madrid, 2003) students need to re-conceptualise their understanding about number to understand multiplicative relationships (Wright, 2011). Multiplicative thinking is distinctly different from additive thinking even though it is constructed by children from their additive thinking processes (Clark & Kamii, 1996). The difference between additive thinking and multiplicative thinking has been characterised by Confrey and Smith (1995) as the difference between a "counting world" and a "splitting world". Essentially a "splitting world" is the ability to share (split) and is an idea to which many students are very sensitive from their earliest experiences (Confrey, 1994). Therefore, this makes splitting part of the multiplicative situation. The position taken in this paper is that multiplication and division are two ways of describing the same situation. In order to help students understand the connection, it may be helpful to represent division as multiplication more often than is currently done. For example, if a simple exercise is considered – If 42 sweets are shared equally among six children, how many does each person receive? – It is suggested that this could be shown as $6 \times \_\_ = 42$, to reinforce the multiplicative situation as an exercise is 'splitting'. The counting world however, identifies additive increments and often 'interferes' with the splitting concept. The counting world does not lead students' thinking into the world of rational numbers (fractions, percentages ratios etc.) in the same way as the splitting world does (Confrey & Smith, 1995).

If teachers are to make explicit the many links and connections within the splitting world (multiplicative situation), and allow students to develop a conceptual understanding of this, rather than focus solely on the Content Strands (Number and Algebra, Measurement and Geometry, Statistics and Probability), they should emphasize the Proficiency Strands (particularly Reasoning and Problem Solving) of the Australian Curriculum: Mathematics (Australian Curriculum, Assessment & Reporting Authority, 2015). Instead of teaching children a set of 'rules' for working with numbers, and teaching ideas like multiplication and division as separate entities, more effective teaching would focus on reasoning about why numbers behave as they do when operating, and understanding how multiplication and division are different ways of describing the same situation. Other curriculums have parallels to the Australian Proficiency Strands. In the United States, the Common Core State Standards for Mathematics (NGA Centre, 2010) contain similar profound statements called Standards for Mathematical Practice. These include a discussion of 'reasoning abstractly and quantitatively' and a description of 'quantitative reasoning' that "entails habits of creating a coherent representation of the problem at hand [and] attending to the meanings of quantities" (NGA Centre, 2010, p. 6.)

This article describes two parts of an on-going study conducted with over 400 primary aged children to determine the extent of their multiplicative thinking. The results of the study are interesting in themselves, however, it is the inferences that can be drawn about teaching and the associated implications for teaching about multiplicative thinking that comprise the main thrust of this article.

Two underpinning aspects of multiplicative thinking were identified by the researchers as being central to conceptual development. First, the concept of equal groups and the multiplicative array are seen as powerful ways of representing the multiplicative situation (Jacob & Mulligan, 2014; Young-Loveridge, 2005) as they enable children to articulate their understanding of the property of commutativity, as well as the inverse relationship between multiplication and division. Second, the language of factors and multiples enables children to explain such ideas (Hurrell & Hurst, 2016). The specific research question on which this article is based is as follows:

- To what extent do upper primary children understand multiplicative arrays, the concept of equal groups, and the language of factors and multiples, and use these ideas to articulate their understanding of the multiplicative situation and related ideas such as the inverse relationship and the commutative property?

## RESEARCH METHODOLOGY

As noted earlier, the research reported upon here is part of a larger current study into primary/elementary school aged children's multiplicative thinking. This article deals with the first phase of the project and the data derived from that phase.

 

### Participant characteristics

The results and discussion which follow are presented in two sections, based on two different parts of the sample. Section 1 involved students from two different schools. School A had an Index of Community Socio-Economic Advantage (ICSEA) ranking of 1082, with a ranking of 1000 being considered as average. School B's ICSEA ranking was 1063. None of the students in the sample from either school was from an English as Second Language (ESL), or Indigenous background and no student in either sample suffered from a disability or specific learning difficulty. Hence the samples could be described as comparable. All parents of the students were provided with information letters about the project. Consent forms were returned for 22 students in School A and 16 students in School B who consequently formed the sample. That is, the researchers included all students (38) for whom a consent form was received. The semi-structured interview instrument was administered with these students. This will be described shortly.

Section 2 involved students from School C which had an ICSEA ranking of 1047. School C had no Indigenous students and 36% of the student population came from an ESL background. Similar to the sample from Schools A and B, no student suffered from a disability or specific learning difficulty. All students in each of the three Year 4, 5 and 6 classes comprised the sample (n=180) and the written Multiplicative Thinking Quiz (MTQ) was administered with these students. This will be described shortly. In each sample from Schools A, B, and C, the participants' classes were taught by a home-room teacher or generalist, not by a specially trained primary mathematics teacher (Hurrell & Hurst, 2016).

### Selection of participants

The principals of the three schools had expressed an interest in their schools being involved in the research project which sought to understand the capacity of primary (or elementary) aged children to think multiplicatively. To begin the project, the researchers planned to gather data solely from Year 6 students but the principals of Schools A and C expressed an interest in involving their Year 5, and Years 4 and 5 students respectively, and the decision was taken to widen the age limits of the sample. This enabled the researchers to identify differences in thinking between different year levels that would inform the later phases of the project.

### Data gathering

There were two instruments used to gather data, a semi-structured interview and a written quiz. Semi-structured interviews were conducted with 38 children in Years 5 and 6 in two different schools (Schools A and B). Interviews lasted between 25 and 40 minutes. A questionnaire was developed from the interview format in order to generate a larger set of data in a shorter time. The questionnaire was administered to nine complete class groups comprising 180 children in Years Four, Five and Six at the third school (School C) and the administration of the questionnaire took about 30 minutes per group. Both the questionnaire and interview were administered to the Year 5 group at School A to establish the consistency of the results from the questionnaire. Whilst the data from the questionnaire was shown to be consistent with those from the interview, richer data were generated from the interview. Burns (2010) asserts that the power of the interview lies in the quality of the question posed by the interviewer or teacher. Examples include "Can you explain how you worked that out?" and "How did you get that answer?" irrespective of whether the child interviewee had the correct answer or not.

Each interview was attended by two researchers, one of whom posed questions while the other wrote anecdotal notes. Interviews were conducted in a quiet open area adjacent to classrooms and lasted around 30 to 35 minutes. In order to provide an opportunity for clarification and deeper reflection on what was said about the mathematics, interviews were audio-recorded for the purpose of later transcription. In order to ensure consistency, the same researcher asked the interview questions on each occasion, using a prepared script. A debriefing session was conducted by the researchers immediately following each interview in which they clarified their thinking and posed questions of each other about participant responses from the interview. This was done to provide a greater level of inter-rater reliability. Both researchers accessed the audio recordings and one of them compiled a detailed analysis of each interview which was then cross-checked by the second researcher (Hurrell & Hurst, 2016).

The written quiz (MTQ) and interview contained the same questions based around key aspects of multiplicative thinking. The strength of the interview lay in the fact that an initial question could be asked and then, depending on the response of the student, followed up with more probing questions. Materials such as counters, bundling sticks, pen/paper, calculator, and Multibase Arithmetic Blocks (MABs) were provided for the interview. Only the interview and quiz questions relevant to the research question stated earlier are included here and shown below.

### Part 1

These questions probe participants' understanding of the multiplicative situation, that one number represents the number of equal groups and the other represents the number in each of the equal groups. As well, the second question seeks to find out if students use a multiplicative array or a number of separate groups to represent a number fact.

- What do the numbers in the number sentence 7 × 6 tell you?
- Please use some of the counters to show what is happening in 4 × 3.
  - Can you show me in a different way?

**Part 2**

This probes participants' knowledge and use of the key mathematical language of factors and multiples and if they can articulate their understanding clearly.

- Can you tell me what a 'factor' and 'multiple' of a number is?
- In 4 × 3 = 12, which numbers are factors and multiples?
- Can you tell me some factors and multiples of 20?

**Part 3**

This section seeks to understand the extent of participants' understanding of the inverse relationship between multiplication and division and the commutative property, and how they articulate that understanding.

- My friend Paul says if you know that 17 × 6 = 102, you can work out the answer to 102 ÷ 6? Is he correct?
- My friend Paul says if you know the answer to 17 × 6, you can work out the answer to 6 × 17. Is he correct?

These questions were chosen because they explore key aspects of multiplicative thinking, the understanding (or otherwise) of which is likely to provide an indication of a student's level of thinking.

Consistent with ethics requirements, all participants were free to withdraw from the project at any stage (Hurrell & Hurst, 2016). The researchers wanted to complete the data gathering process promptly to avoid any potential validity issues with the data such as 'maturation threats' or 'history threats' (Fraenkel & Wallen, 2003). Similarly, no more than four interviews were conducted on any given day to avoid other potential threats to validity such as 'instrument decay' (Fraenkel & Wallen, 2003; Hurst & Hurrell, 2016). Due to the fact that the sample size is relatively small, it is not possible to make substantial generalisations about children's knowledge. Rather, the purpose of this article is to point out some aspects of some children's thinking and the inferences that might be drawn from them. As noted the research reported upon here is part of a current and larger project, one aspect of which includes the interviewing of a larger cohort of students to see if such generalisations might be possible.

## RESULTS AND DISCUSSION

The responses from the interviews and questionnaires made for some interesting overall observations. First, responses from the Year Five cohort at School A and the Year Six cohort at School B revealed a wide range of conceptual understanding. Second, responses to the questionnaire administered at School C revealed that the three class groups within each year level had varying levels of understanding. While there were large variations within each year level, a similar range was evident between year levels, and indeed, within each class group. This paper suggests that the differences may have resulted, at least to some extent, from different pedagogies, teaching styles, and/or may reflect different stages of development of children's understandings of multiplicative concepts. After all, Siemon et al. (2011) have noted that multiplicative thinking usually does not fully develop until the early secondary years.

### Section 1: School A and School B

The purpose of this paper is not to compare performance of different school cohorts against one another or different sections of school cohorts against one another. Rather it seeks to identify aspects of multiplicative thinking that might be evident or otherwise in different children and to understand why that might be so. Hence interview results from Schools A and B are combined into one set. **Table 1** presents a summary of responses to the five illustrative questions listed above for the Year Five cohort from School A and the Year Six cohort from School B (n = 38).

It is also worth noting that of the 38 children in the combined sample, ten (26%) responded correctly to four or five of the above questions and a further seven (18%) responded correctly to three of the questions. This seems to indicate that approximately one quarter of the sample demonstrated a strong level of conceptual understanding of the selected aspects of multiplicative thinking and a smaller proportion showed a reasonable level of understanding. However, over half the children in the sample could only respond appropriately to two or less of the selected questions. This points to a wide range of understanding across the sample. These findings are congruent with the percentages of students who struggle with multiplicative thinking (Clarke & Kamii, 1996; Siemon et al., 2006), as described earlier.

**Table 1.** Summary of Responses from Schools A and B

| Mathematical understanding demonstrated by responses to listed questions | School A Year 5<br>School B Year 6 |
|---|---|
| Identifies numbers in multiplication fact as 'group size' and 'number of groups'. | 39% |
| Represents given multiplication fact as an array. | 34% |
| Defines 'factor' and 'multiple' and/or identifies factors and multiples in given number fact. | 63% |
| Explains commutative property in a conceptual way and/or demonstrates it using an array. | 29% |
| Explains inverse relationship in a conceptual way based on number of groups and group size | 50% |

**Some typical strong responses**

Typical responses demonstrating a strong level of conceptual understanding of the commutative property of multiplication include the following:

Student Dylan – "It doesn't really matter which way it is – seventeen groups of six is the same as six groups of seventeen". He then used tiles to make three groups of five and five groups of three, and also rearranged twelve tiles saying "I just put them into a three by four grid – it's the same as a four by three".

Student Dean – "It's just the same . . . you just flip it around". He then used tiles to make a three by five array and rotated the array to explain his point.

Similarly, the following exchange during the interview with Student Jason shows some connection of ideas around the inverse relationship between multiplication and division, sharing into equal groups, and arrays.

When discussing the division fact 24 ÷ 3, Jason showed it as an array and said, "Then I'm going to split it up into threes, because I'm going to see how many groups of three I can have in 24". Also said, when asked what the answer would be, "I started with knowing that how many threes go into 15 and that's five, then I counted by threes to get 18, 21, 24". He also said, "If I had 3 times 4 it would be 12. If I had 12 divided by 3 it would be 4". He also gave a similar example with "8 groups of 3 = 24, so 24÷8 = 3"

Such connected discussion seems to demonstrate a sound understanding of the concepts involved.

**Responses indicating partial understanding**

The apparent lack of conceptual understanding in the responses of some children is of interest. It is difficult to draw conclusions about the depth of some children's conceptual understanding given the absence of links and connections between responses to different questions. That is, some children show some understanding of a particular idea which might lead one to reasonably expect they would show an understanding of related concepts. However, this was often not the case.

It is well accepted that the array is a powerful representation of the multiplicative situation (Jacob & Mulligan, 2014). However, while two of the children (Ellie and Tilly) drew an array to represent the given number fact, neither of them could explain why the commutative property works, in terms of the array. Rather, they said that the numbers were 'swapped around' (Tilly) or 'you've just swapped them around' (Ellie). Tilly and Ellie and other students like them could not articulate why you could 'swap' the numbers around, and it appeared that they had developed a procedural, perhaps even rote, response to the question, rather than one with an underlying conceptual understanding. Also, of the thirteen children who drew an array, only seven of them could describe factors and multiples.

Similarly, while 63% of the children (n = 24) could adequately describe factors and multiples and their roles in the multiplicative situation, only five of them talked about the 7×6 number fact in terms of group size and number of groups. Further to that, 39% of the children (n = 15) described the number fact in terms of group size and number of groups, yet only five of that group also drew an array. Some of the children (n = 11) adequately explained the commutative property and half (n = 19) explained the inverse relationship between multiplication and division. However, not all of the eleven children who explained the commutative property could also explain the inverse relationship. This is interesting because the ideas underpinning those interview tasks are inextricably linked – that is, group size/number, the factor-factor-multiple relationship, the representation as an array, the commutative property, and the inverse relationship. Hence, it might be reasonably expected that there would be more children who could perform well on all or most of the items, or on none (or very few) of them.

**School A and B Implications**

The inferences that can be drawn from the School A and B data suggest that the connections between those important ideas need to be made clear and more explicit so that a mutual understanding of them can be developed. This is supported by the observations that: some students drew an array; some others could explain factors and multiples; and some others could explain a multiplication fact in terms of group size and number. Perhaps it is because there had been passing mention made of these key ideas, rather than sustained and explicit teaching of

 

**Table 2.** Comparison of Responses from Different Year Levels at School C

| Mathematical understanding demonstrated by responses to listed questions | Year 4 | Year 5 | Year 6 |
|---|---|---|---|
| Identifies numbers in multiplication fact as 'group size' and 'number of groups'. | 14% | 9% | 23% |
| Represents given multiplication fact as an array. | 28% | 32% | 39% |
| Defines 'factor' and 'multiple' and/or identifies factors and multiples in given number fact. | 3% | 15% | 14% |
| Explains commutative property in a conceptual way and/or demonstrates it using an array. | 0% | 2% | 5% |
| Explains inverse relationship in a conceptual way based on number of groups and group size | 10% | 1% | 12% |

**Table 3.** Comparison within Year Levels in School C

| Mathematical understanding demonstrated by responses to listed questions | 4A | 4B | 4C |
|---|---|---|---|
| Identifies numbers in multiplication fact as 'group size' and 'number of groups'. | 0% | 6% | 35% |
| Represents given multiplication fact as an array. | 62% | 17% | 0% |
| Defines 'factor' and 'multiple' and/or identifies factors and multiples in given number fact. | 0% | 6% | 5% |
| Explains commutative property in a conceptual way and/or demonstrates it using an array. | 0% | 0% | 0% |
| Explains inverse relationship in a conceptual way based on number of groups and group size | 14% | 0% | 15% |

them. For example, the fact that less than one third of the children could explain the commutative property in a conceptual way gives rise to questions about how the commutative property may have been taught. Perhaps it is also attributable to the fact that children's understanding of the multiplicative situation is developing and in a state of flux. After all, it has been noted (Siemon et al., 2011) that multiplicative thinking is a concept that does not fully develop until around the age of fourteen and the students involved here are several years younger than that.

The lack of sustained teaching may also be because the teachers simply do not appreciate the critical importance of the ideas of factor/multiple, group size/number, and the use of the array. Hence they may have taught some of the ideas once, assuming that such limited exposure was appropriate when the evidence would suggest otherwise.

**Section 2: School C**

In School C, the questionnaire was administered to 180 children in Years Four, Five and Six. **Table 2** represents the responses from the three year levels in School C to questions about the same concepts as shown in **Table 1**.

In general, it would probably be expected that the Year Six children would perform better than the Year Five children who would in turn perform better than the Year Four children. However, as can be seen, this is not always the case and even where it is, one would perhaps expect the comparative performance of the older children to be markedly better than it is.

Of more interest is the comparison within each year level in School C, as shown in **Table 3** which shows responses from children in the three Year Four classes. Here, there are marked differences in the responses from different class groups, particularly in relation to the first two questions.

It is noteworthy that no child in Class 4A could identify 'group size' and 'number of groups' in multiplication facts when over a third (35%) of Class 4C could do so. Even more intriguing is that nearly two thirds (62%) of Class 4A drew an array to show a multiplication fact when not one child in Class 4C did that. As well, very few children in Class 4B responded correctly on any of the five questions. What might this indicate?

**School C Implications**

Classes at School C are not streamed on ability. Hence, it seems reasonable to assume that the variation in responses may be due to different teaching occurring among the three Year Four classes. Perhaps there has been a clear emphasis in Class 4A on the use of arrays, rather than showing multiplication facts as separate groups. It is also worth noting that the responses from Class 4A (62%) to the array question were the highest of any class in the school – only one Year Six class (53%) and one Year Five class (50%) recorded a similar level of correct responses. Similarly, the teaching in Class 4C is likely to have emphasized the notion of 'group size' and 'number of groups' in the multiplicative situation. Again, Class 4C's response (35%) is the highest recorded of all classes with only one Year Six class (33%) recording a similar level of correct answers. What is not apparent across these classes, nor across Schools A and B, is explicit, sustained teaching of the connections between the five related ideas in the multiplicative situation.

 

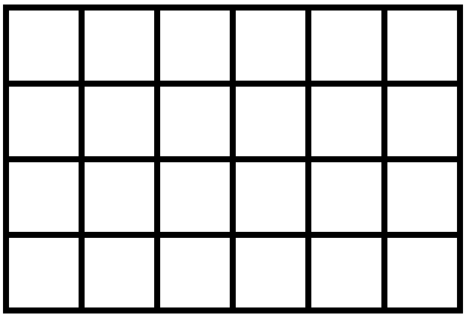

**Figure 1.** Multiplicative array

## MULTIPLICATIVE THINKING: WHAT HELPS STUDENTS?

If, as the research tells us, multiplicative thinking is vital for further success in mathematics, but difficult to learn, then teachers need the content and pedagogical knowledge to succeed in their endeavours to effectively teach it. Carroll (2007) has constructed a list of strategies and ideas that support multiplicative thinking, as well as some pedagogical issues about which teachers need to be aware.

- Allow children to work out their own ways to solve problems involving multiplicative thinking.
- Compare additive and multiplicative thinking approaches.
- Use models that clearly illustrate the idea/s.
- Sometimes students are introduced to the ideas symbolically before the groundwork has been done to establish meaning and become comfortable in working with them.
- Make and discuss the links between fraction ideas, rates, ratios and proportion.
- Use authentic contexts and models to exemplify situations.
- Estimation is really important as it demonstrates understanding of the concepts involved.
- Engage in conversations about the ideas and talk about the links, discuss the similarities and differences between the ideas.
- It is development of fuller, deeper and more connected understandings of the number system that makes a difference.

Carroll (2007, pp. 41 - 42)

In the remainder of this article we would like to pursue dot point three of Carroll's (2007) list; "Use models that clearly illustrate the idea/s". Although this will be the focus, it should be noted that by carefully considering the model used to 'carry' the understanding, at one time or another, the remainder of Carroll's list should be exercised.

## A MODEL FOR THE DEVELOPMENT OF MULTIPLICATIVE THINKING

One model for trying to build a conceptual understanding of multiplication is the multiplicative array. Multiplicative arrays refer to representations of rectangular arrays in which the multiplier and the multiplicand are exchangeable (**Figure 1**). These arrays are seen by some as powerful ways in which to represent multiplication (Barmby, Harries, Higgins & Suggate, 2009; Young-Loveridge & Mills, 2009). Young-Loveridge (2005) asserts that multiplicative arrays have the potential to allow students to visualise commutativity, associativity and distributivity. Further, Wright (2011) states that multiplicative arrays embody the binary nature of multiplication, and that as a representation, they have value, as they also connect to ideas of measurement of area and volume and Cartesian products. We will visit the use of multiplicative arrays in the activity called "A bag of tiles."

### Task - A Bag of Tiles

This task can be varied to suit the teaching and learning of several aspects of multiplicative thinking. Essentially it is based on students working with a set of 2cm × 2cm plastic tiles. These can be given out in a plastic snap-lock bag and can vary in number, depending on the task and the targeted aspect of multiplicative thinking. However, for the following activity each pair of students is given 24 tiles. Although each student could be given their own set of tiles, if we want the students to engage in meaningful conversations about the task and about the multiplicative thinking behind the task, then having the students in pairs is actually a more productive setting.

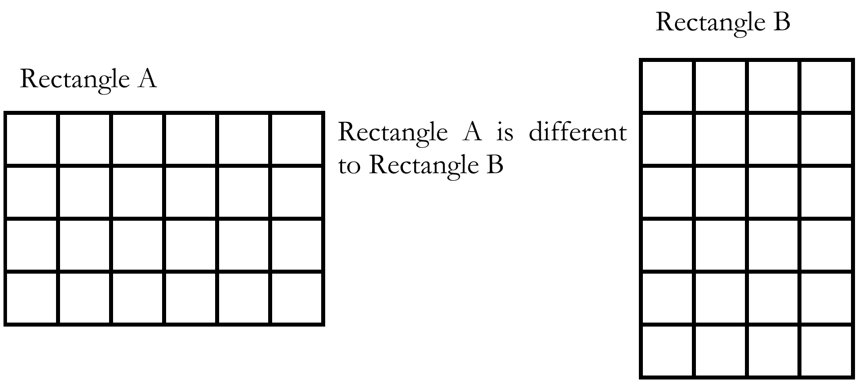


**Figure 2.** Two "different" rectangles, 6×4 and 4×6

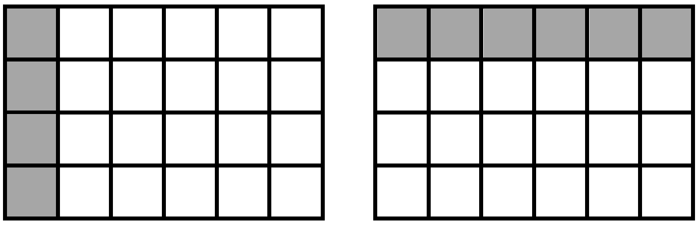

**Figure 3.** Overlaying Rectangle A with different strips

The basic task is for children to make an array with the tiles so that rows and columns contain the same number of tiles with none left over. In this case one array which might be produced is a 6 × 4 configuration. The choice of 24 is worthy of note as it can also result in arrays of 8×3 or 3×8), 12×2 (or 2×12), and of course 24×1 (or 1×24). The different arrays A and B (**Figure 2**) provide an interesting discussion point in leading children to a realisation that although the two arrays arrive at the same total, the manner in which they are constructed is important in certain contexts. For instance, there may be very big ramifications in not understanding that, although in a day you would end up taking twelve tablets, taking two tablets, six times a day may have very different effects from taking six tablets twice a day. This idea can be further developed by asking children to 'tell a story' about each number fact to show that 4×6 (four rows of six) is indeed different to 6×4 (six rows of four). The notion of 'story telling' invokes Carroll's (2007) points regarding discussions and conversations, and authentic contexts.

As well, the two arrays offer a good opportunity to develop an understanding of the commutative property in a deeper way that by simply rotating the array. Different coloured strips of four and six squares can be laid over the array to show that while the two arrays represent the same product, they are in fact different (**Figure 3**). That is, four yellow strips of six cover the array in the same way as do six green strips of four. The result is the same but the situation is different and this is the essence of understanding the commutative property.

The students are then asked to find all of the different rectangles that can be made from 24 tiles. The bag of tiles activity works very well as a physical representation to develop an understanding of factors and multiples as well as the commutative property of multiplication, both very powerful understandings which will be often called upon in mathematics. This is also a good opportunity to make connections between the representations of the arrays and the way we symbolically record them. Further we can exploit the opportunity to discuss and show the links within the multiplicative situation (i.e., the inextricable relationship between multiplication and division, being, 6 × 4 = 24, 4 × 6 = 24, 24 ÷ 6 = 4 and 24 ÷ 4 = 6, as well as the division construct for fractions). Over and above the mathematical content that this activity contains, it also embodies the four proficiency strands (Problem Solving, Understanding, Reasoning and Fluency) as articulated in the Australian Curriculum: Mathematics (ACARA, 2015). There are also clear links to the Common Core State Standards for Mathematics where the Standards for Mathematical Practice include discussion of the notion of "looking for and making sense of structure" (NGA Centre, 2010, p. 6).

Further to the richness that can be gleaned from using 24 tiles and asking the questions that are articulated above, the same activities can be entered into with arrays of other sizes to further develop and re-inforce the understandings. The tiles can then be employed to investigate prime, composite, square and triangular numbers.

For example, to develop an understanding of prime and composite numbers, the students are given more tiles to work with, and are instructed that they cannot rotate the tiles (employ the commutative property) and still consider them to be 'different'. Therefore, a 6 × 4 configuration is considered to be the same as a 4 × 6 configuration. They are then asked see if they can make a rectangle with two tiles in more than one 'different' way? They build the array and record the finding that the only configuration for two tiles is a 2 × 1 array. The students

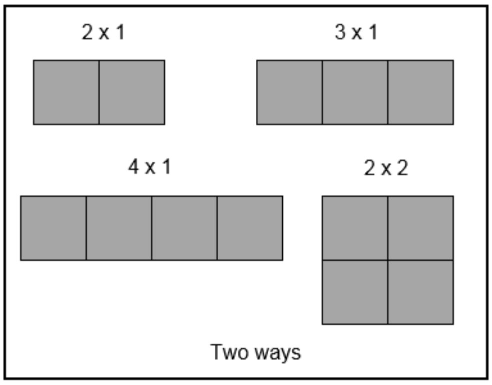


**Figure 4.** Arrays for 2 tiles, three tiles and four tiles

then investigate three tiles and continue their investigations as required, but probably for not less than the 24 tiles with which they started (**Figure 4**). There is a conversation to be had with first of all, four tiles, and then with nine tiles, and then 16 tiles (possibly even 25 tiles) about whether a square is a rectangle.

Initially what the students are making and recording are the factors for whole numbers between two and 24. What is also occurring, is an opportunity to talk about the numbers for which two or more sets of factors cannot be found (prime numbers) and the numbers which have multiple sets of factors (composite numbers). The exploration here of course is what makes some numbers prime numbers and others, composite numbers. Also, by previously considering arrays for the numbers four, nine, 16 and perhaps 25, an exploration of square numbers and why we call them square numbers can be undertaken.

## GENERAL IMPLICATIONS

The five selected questions from the interview and questionnaire represent less than a quarter of the full instrument yet the data generated from just three sets of children have provided plenty of food for thought. There are two main observations that can be made from the presented data. First, there are considerable differences in the levels of understanding of multiplicative concepts shown by two groups (Schools A and B) that were interviewed. Some children displayed more connected understanding than did others. Second, there is considerable difference in responses among classes in the same year level at the same school (School C) where the questionnaire was administered. In seeking reasons for this, it is reasonable to infer that the differences may be due to pedagogies.

The differences in responses are quite stark at times and the relative connectedness in the thinking of some children in the combined cohort from Schools A and B suggests that connections between ideas may have been made more explicit in some classes compared to others. This raises the question of the level of opportunity that students have in their classroom to engage in problem solving and reasoning when developing multiplicative thinking and perhaps other areas of mathematics. Also, in responding to questions other than those reported here, some children were reluctant and/or unable to depart from quite procedural responses which was not the case with other children.

## CONCLUSIONS

In this article we have only just begun to explore the opportunities afforded by multiplicative arrays to support students in understanding the multiplicative situation. Arrays can also be used to link to the division construct for fractions – e.g., a group of twenty four tiles can be split into quarters so that one quarter of 24 is 6, two quarters of 24 is 12, three quarters of 24 is 18. The same can be said about sixths. Further the concept of why we calculate area as we do can be explored by overlaying the tile array with a clear grid of the same number of squares (i.e., for a 4 × 6 array, use a clear grid of 2 cm squares in a 4 × 6 pattern). Students are asked to describe the area of the grid.

In conclusion, there are implications for teaching in terms of what can be done to help children develop key multiplicative concepts in a connected way. The evidence presented here suggests that such pedagogical practices exist but may not be sufficiently widespread. Such teaching could include the following:

- Explicitly teach that the multiplicative situation is based on the number of equal groups and the size of each group.
- Develop an understanding of the terms factor and multiple through the use of arrays, and explicitly use them as 'mathematical language'.

- Teach multiplication and division simultaneously, not separately.
- Develop the commutative property through the use of arrays and physically show the 'x' rows of 'y' gives the same result as 'y' rows of 'x'.
- Develop a rich understanding of the multiplicative situation through the proficiencies of problem solving and reasoning.

If teachers view multiplication and division as different ways of representing 'the multiplicative situation', rather than as separate entities, the links and connections between the ideas discussed in this paper can be made explicit for children. When those connections are clearly understood, ideas such as the inverse relationship and the commutative property become much easier to grasp. The multiplicative array is a powerful tool for developing those connections.

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# GeoloGIS-BH: An Information System for Using the Built Heritage for Geological Teaching

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## ABSTRACT

There are examples of using stones of the cultural heritage for teaching purposes. Information systems have found several potential uses in the promotion and preservation of cultural heritage. In this paper is considered the conceptual framework of an information system concerning features of geological interest (FGI) in the built heritage (without any consideration in terms of its software implementation). This FGI concept is used here in a very wide sense to encompass characteristics of geological materials that can be recognized with the naked eye and analogies of geological processes in the built environment. Two perspectives are considered for information organization: occurrences of FGIs in the built heritage (more suitable for Earth Sciences teaching) and FGIs as components of built heritage elements (more suitable for humanities teaching). The main issue that arises from the ensuing discussion was found to be the findability of a given FGI, depending on its visual contrast and the characteristics of the built heritage element. It is argued that, in this way, geological concepts can contribute to the promotion and conservation of the built heritage.

**Keywords:** built environment, feature of geological interest, Earth Sciences teaching, information systems, spatial referencing

## INTRODUCTION

Stones in buildings can be important elements for geological illustration (some examples of this can be seen in Williams 2009, Williams 2012, Pereira & Marker 2016) and its weathering can also be used as it relates to environmental conditions (Perez-Monserrat et al., 2016). There are several examples of using GIS for preparing and managing teaching and touristic activities in relation to cultural elements (Balestro et al. 2015; Hoerig et al. 2015; Sheng & Tang, 2015).

However, we were unable to find any previous example similar to the perspective considered in the present paper: the use of spatial-based information systems for promoting building stone use in education (formal and informal - this last one being similar to tourism promotion).

We propose that this distinction is a non trivial one as there will be specific problems related to it that, hopefully, we will be able to show here. A brief abstract in Portuguese and the corresponding presentation also in Portuguese in relation to the specific subject of this paper have already been presented (Alves et al. 2016).

## THE GEOLOGIS CONCEPTUAL FRAMEWORK

The basic principles of the GeoloGIS (originally SIGeolog in Portuguese) conceptual framework have already been presented in Portuguese (Cunha et al. 2016). Hence, we will present here a brief synthesis of it considering the features that are relevant to this paper. However, we will not consider any question in relation to the software implementation of the system. The GeoloGIS is presented as a system made by sets that we will call sections (corresponding to geologic themes such as Petrology, Stratigraphy, etc.) with diverse kind of objects and that (at least at this conceptual level) can be seen as an extended version of the classical “layer” concept without limitations in terms of the characteristics of the objects, which, in any given section can be of matrix or vector type, points, lines, areas, surfaces, volumes and even objects of a higher dimension (objects of n-dimension, in general).

One of the premises of this structure is that it must be flexible and allow its information to be re-organized for new applications. We will illustrate this with the extension of the section of geologic materials to the promotion of the geological features of the cultural heritage.

Each object on a given section will have a matrix of informations in relation to diverse characteristics. This informations can be of diverse types such as nominal, ordinal, numerical and even objects which can have informations of the types mentioned, defining an open system (similar to the folder system in the popular operating systems such as Windows or Ubuntu). The system sections should be able to share objects and their informations. The system must be able (this is critical for the present work) to create new sections by copy, redistribution or synthesis of existing informations, or by addition of new ones.

The extension of the GeolGIS framework to the geological component of the built heritage will be discussed in the next section.

## TWO PERSPECTIVES ON DATA ORGANIZATION

The GeoloGIS-BH will be discussed as a collection (that could be used autonomously) prepared from the collections of the basic GeoloGIS, such as Petrology, Geological Resources and Engineering Geology. Two perspectives of data organization will be considered (**Figure 1**), considering the dual purpose of showing, on the one hand, to science students examples of the application of their study objects and, on the other hand, showing humanities students characteristics of the constituents of their objects (these two perspectives can constitute different applications that could be obtained from the same basis following the procedures presented in the previous section). The first one (more suited to a science-minded public, specially Earth Sciences students but also for public with interest in biology, physics, chemistry) sees the built heritage elements as occurrences of examples of materials, their characteristics and transformations as well as their applications, what will be referred generically as features of geological interest (FGI); the objects will be the FGI; the built heritage places will be part of the

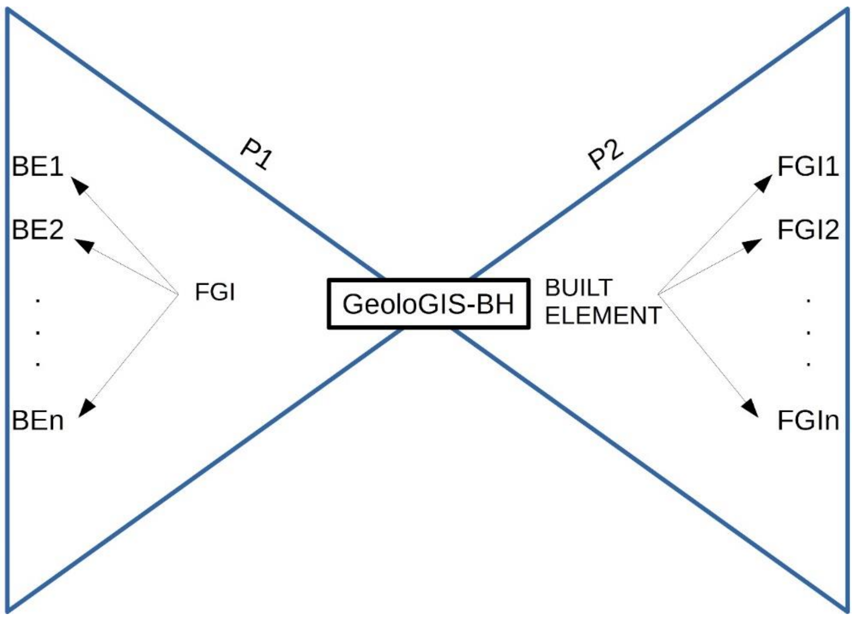


**Figure 1.** A visual comparison of the two perspectives proposed for the GeoloGIS-BH: in P1 (left side), the feature of geological interest (FGI) is the centre of attention and are registered the built elements where it can be found (BE1 … BEn) while in P2 (right side), the built element is the centre of attention and are registered the features of geological interest that are present on it (FGI1, …, FGIn).

**Figure 2.** Examples of features of geological interest (FGIs) in built environment: biotitic porphyric granite (upper left hand); twinning on feldspar (upper right hand); aplite-pegmatite textures in a veinlet of a two-mica granite (lower image).

information on those FGIs. In the second perspective (more suited for a more historical and architectural orientated public), the built heritage elements are the objects and the geological materials are seen as informations of these objects.

In the first perspective, as already said above, the main focus will be on the FGIs that can illustrate (and be teaching tools) for rock types, textural and structural features, weathering transformations and other geological features. The cultural heritage places will serve the spatial location of these illustration/teaching FGIs. The FGI concept is used here in a very wide sense that includes features in geologic materials that can be recognized in the field with the naked eye as well as analogies of geologically interesting processes that develop in the built environment without intervention of humans (both in geological materials or in materials prepared from geologic raw materials). For example, and following the illustration of already existing applications on other areas such as hotel search, one can occurrences for places in a given location (administrative unit) or in a given radius of a reference point that show volcanic rocks, tourmaline in veins, weathered granites, carbonate rock erosion or "mineral"-like neoformations. In **Figure 2** are presented some examples of the use of built elements for showing geological features such as mineralogical and textural features: in **Figure 2** (upper left image) is shown a porphyric biotitic granite, in **Figure 2** (upper right image) is shown an example of twinning in feldspars and in **Figure 2** (lower image) is shown the occurrence of aplitic-pegmatite veinlet in a two-mica granite. The built environment

**Figure 3.** Example of calcium carbonate neoformation with stalactite-like structure on a granite slab (if you look closely you might be able to see on the lower tip of the "stalactite" a drop of the solutions that are forming it).

can also be used to show geologic substances (and their features) not available locally, e.g. carbonate rocks on granite locations (as will be illustrated in the next section).

However, care must be taken in relation to this use of features in the built heritage as they occur outside their original geological context constituting an extreme case of censored observation. It can be argued that all geological observations are censored observations (limited to the available outcrops, boreholes or indirect geophysical and geochemical information) but in the case of the materials in the built heritage knowledge improvement is severely limited. For example, one frequently does not even know whether adjacent blocks came from the same quarry and there could be human interventions at different times.

The question of crystalline substances neoformations in the built environment might be worthy on some further comments as it can be a polemic point (see Alves, 2013a). According to the International Mineralogical Association (IMA) recommendations (Nickel, 1995), inorganic crystalline substances that result from interaction of human materials with the weather elements are not considered minerals since this can create a diversity of substances that does not correspond to the natural environment. However, neoformations in the built environment generally are also found in the natural environment (Alves, 2013a,b) and since there were not made purposely by humans we think it will not be against the spirit of these IMA recommendations to consider the neoformations of the built environment as minerals. A possible exception to this could be the weathering products of metals or advanced (non-traditional) materials. However, these products are not totally devoid of interest for the geological sciences as they can suggest the conditions where new minerals could be found.

The neoformations of the built environment can be useful to illustrate morphological, textural and, depending on the available laboratory conditions, chemical and internal structure features. For example, a common feature on the built environment are occurrences of calcium carbonate deposits with a calcite structure that in some places (depending on the spatial patterns of the solutions circulation) can produce structures similar to stalactites (as in **Figure 3**) and stalagmites. It is possible, and the first author of this paper has performed that experience with students, to perform simple field tests such as pH paper tests of the solutions, showing their alkaline character (see illustration in Alves and Sanjurjo-Sánchez 2015) or acid reaction of the carbonate deposits. It is also possible to collect samples from the carbonate deposits for optical microscopy (e.g. to show deposition textures), X-ray difractograms or scanning electron microscopy studies. Another experience concerns the sampling of solutions from the places where these substances are being deposited for simple experiences such as laboratory crystallization by simple air drying (the crystallization products can be studied with simple optical instruments or can be subject to more advanced laboratory studies).

However, the genetic context of the neoformations deserves some careful framing as there are some substances that occur in context very different from the natural one, as is the, rather common, case of gypsum crusts on granites (Sanjurjo-Sánchez et al. 2009). There are also situations where while there are great similarities in terms of texture and structure, the chemical basis of the genesis process present significant differences, as in the case of the carbonate deposits (see Liu & He, 1998).

 

Figure 4. Illustrative image of relations between substrate and biological colonization with lichens and moss on the stones and plants in the joints between the wall stones.

Figure 5. Image of two granite types on columns of a portal in a monument of Braga (NW Portugal): the darkish-brownish one on the right is similar to the local granite while the lighter one on the left is similar to certain *facies* from surrounding regions.

The built elements can also contribute to the teaching of relations between substrates and biological activity as is illustrated in Figure 4 where the development of biological colonization shows patterns associated with the type of substrate with lichens and moss on the stones and plants on the spaces between the stones.

This first perspective might include historical and architectural (and engineering) information on the places where the geologic materials are applied as they relate to the geological context and the characteristics of the materials but these components will be dominant in the second perspective. It can be considered that this second perspective will be stretching too far the concept of the GeoloGIS given the dominance of non-geological information but we argue that the information of the basic GeoloGIS on geologic materials can be organized to be integrated in this second perspective, constituting an example of collaboration between the hard sciences, such as Geology, and the humanities. In this second perspective the level of detail for the FGI will be generally lower and findability will be higher since usually the reference is to an architectural element with FGIs such as a stone or a stone group of a rock type or diverse rock types as in Figure 5 where it is intended to show two different types of granites on a portal (one of which is local while the other not). However, there could be more specialized situations (for example for post-graduate students of the humanities areas), where it is intended to show FGIs such as a given mineral occurrence that supports a certain hypothesis concerning the source of a given material or FGIs that can be used for dating heritage elements (Sanjurjo-Sánchez, 2016).

**Figure 6.** Image from a granite fountain (location of specific details will be an excruciating challenge).

## FEATURES FINDABILITY

Besides the software implementation issues, the main problem for this extension of the GeoloGIS will be related to the question of "ambient findability" (Morville 2005), to wit, the development of reference systems to locate the occurrences of the places for observation of the FGIs, especially for the first perspective. Of course the question of the findability of a given FGI will also depend of its visual contrast (resulting from size, colour and texture in relation to the surrounding), as is illustrated in **Figure 5** where the granite types have a clear colour contrast that helps their distinction but in the following discussion this question will be ignored (i.e. we will treat the subject of location regardless of visual contrast).

The complexity of this issue will range from the observation of a given rock type on free standing monolithic structures (as illustrated in **Figure 2**, upper right image) where GPS coordinates and a photograph might be enough (the structure is the FGI). However, findability will markedly decrease in the case of a specific FGI that occurs on a given spot. In the case considered one can add some descriptors like south view, height from ground and horizontal distance from a given side. There are elements (as illustrated on **Figure 6**) that might be easy to locate (by GPS or by building plan) but due to their geometric round form and irregular distribution of decorative it will almost impossible to define a referencing system for defining the position of more spot-like FGIs (weathered features; rare mineral occurrences). In such situations tools like PhotosynthTM (https://photosynth.net/default.aspx) from Microsoft Corporation can help in the visual location of features.

Our next example concerns the Youth statue (**Figure 7**) in downtown Porto (NW Portugal); a town located on granite terrains. The statue is on a squared base pedestal whose sides are made of two carbonate rocks types: a low porosity limestone with fossils (as shown in the inset at the low left of **Figure 7**) and marble. In this case the monument can be located by GPS and the location of specific details can be easily established by geometrical referencing for the stones in a given face of the pedestal (in the case of **Figure 7** it is the east side) but it will be harder for the more irregularly disposed stones (and their weathering features) on the water basins at the bottom of the pedestal.

Figure 7. View of the east side of the pedestal of the Youth statue in Porto (NW Portugal – a town located on granite terrains) with inset showing the fossils.

Figure 8. A wall with regular distribution of regular shaped stones (it is possible to define referencing for locating individual stones and individual features inside a given stone).

Regular walls (as in Figure 8) that show diverse types of granitic rocks) made of regular geometrically stones can be also object of precise referencing (due to the geometrical conditions). The wall can be locate by GPS coordinates (or, in this case, by a building plan), and the regular pattern allows to locate each individual stone in the wall (by indicating the line number from upper or lower one and the stone number in that line from either right or left or south or north). The geometry of the stone faces (rectangles) allows locations of smaller FGIs within each stone by using horizontal and vertical distances from reference points (e.g. upper left corner). In this way it is possible to "digitize" the geological features of the wall stones.

Rock aggregates (that can be a teaching tool, either in terms of basic concepts such as rock types or in terms of rock properties) illustrates the issue of referencing: it will be easy to show the use of a rock type (e.g. see pavement in street X) but it might be very difficult (impossible?) to locate a FGI in a specific aggregate particle.

 

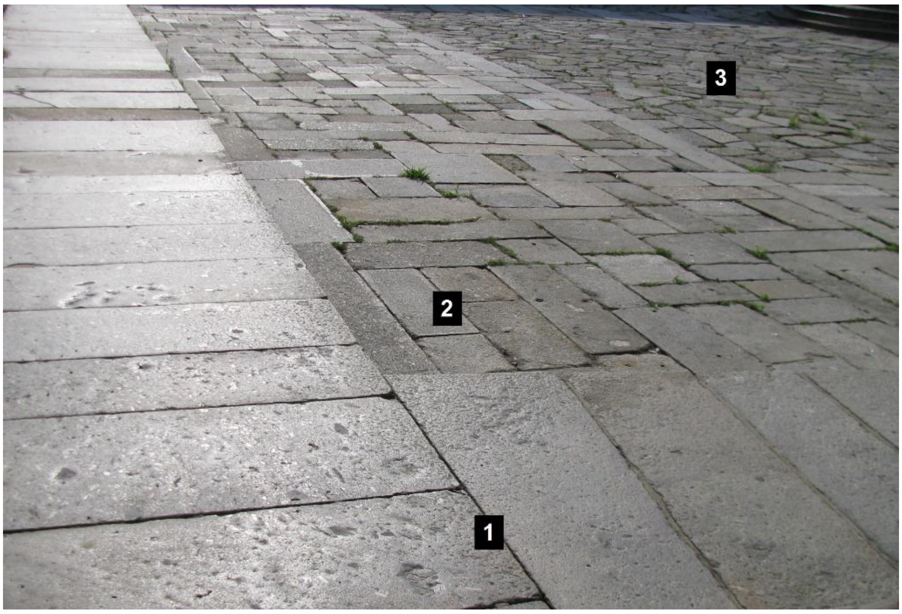


**Figure 9.** Illustration of referencing situation in pavements: regular disposition of geometrically regular stones in 1, geometrically regular stones disposed in a more uneven manner in 2 and irregularly shaped stones disposed in an uneven fashion.

The image presented in **Figure 9** illustrates (in a single image) different situations in pavements: in the areas marked as 1, there are regular stones disposed in a regular pattern and, hence, locating a stone or a feature within a stone will be straight forward (as in the example of **Figure 8**). In the area marked as 2, the individual stones still show a regular geometrical pattern but their disposition is irregular and it will harder to locate a given FGI (whether a stone or a feature within a stone). The areas marked as 3 constitute the worst scenario for FGI location as the individual stones and its disposition are irregular. Steps on traditional stairs will be also an example of regular pattern of regular stones but some extended stairs (several stones by step) and specially rounded versions of stairs will make location more difficult.

Geocaching activities (as discussed, e.g., in (Lo, 2010) can be a fun, hence motivating, way to turn the tables on this question of locating geological features on the built environment and search for creative procedures for location with the added (motivating) challenge of not being solved just by GPS coordinates. Perhaps there might be here an opportunity for promoting research in referencing systems.

## FINAL CONSIDERATIONS

As has been shown in the previous sections, the built heritage present diverse opportunities for Earth Sciences teaching (at several levels of formal education, including post-graduate studies, as well as in more informal contexts including tourist activities) concerning what we referred as feature of geological interest, or FGI for short, considered here in a very wide sense that includes characteristics of geological materials that can be recognized with the naked eye and analogues of geological process on the natural environment such as crystalline neoformations, erosion figures and illustrations of the relations between substrate and biological colonization. However, this approach requires some care in terms of teaching in relation to the occurrence of geological materials outside their original geological context (as this will be extremely censored observations) and, in the case of analogies, regarding the differences between the genetic processes in the natural and built environment.

These FGI can be integrated in an information system referred here as GeoloGIS-BH (BH-built heritage). In terms of information organization, two perspectives were considered that might be of interest for different target audiences. In one of them, the GeoloGIS-BH is seen as a collection of FGIs located in built heritage occurrences. This will suitable for Earth Sciences teaching (and might also appeal to students of other sciences). In the other perspective presented above, more oriented to humanities teaching, the FGIs are components of built heritage elements and their interest will be related to their historical and architectural information content, i.e., whether they give information in historical issues and architectural issues such as the selection among local materials or the importation of non local materials due to aesthetic and functional reasons.

Besides the questions concerning the implementation as software of this system (not considered here), the main conceptual issue regarding the GeoloGIS-BH will be the findability (spatial location) of a given FGI which will depend on its visual contrast resulting of color and texture in relation to the surrounding areas and the size of the

 

FGI in relation to the built heritage element as well as on the geometrical regularity of the built element where the FGI is found. The question of findability will be, in general, more relevant for the first perspective referred above, as in the second the discussion of geological features will be at a coarser scale, but in the case of a finer detailed FGI (e.g. for discussing materials provenance), the question of findability might rise again.

In this way, geological concepts can be used for the promotion and valuing of the built heritage through roadmaps showing occurrences of certain features or as part of information for a given monument. These perspectives might even give a contribution for the justification of conservation procedures, i.e. the conservation of a given built heritage element for showing a rare FGI, either locally or even globally.

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# A Comparative Study of Geometry in Elementary School Mathematics Textbooks from Five Countries

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## ABSTRACT

The purposes of this study were to compare the differences in the use of geometry in elementary school mathematics textbooks among Finland, Mainland China, Singapore, Taiwan, and the USA and to investigate the relationships between the design of the textbooks and students' performance on large-scale tests such as TIMSS-4 geometry, TIMSS-8 geometry, and PISA space and shape. The content analysis method was used to collect data, and then chi-square tests and correlation analyses were used to analyze data. The results showed that there were significant differences in representation form, problem type, and question format among these mathematics textbooks from the five countries. Moreover, the strength of the positive relationships between visual form (combined form) and students' performance on TIMSS-4 geometry, TIMSS-8 geometry, and PISA space and shape decreases as students advance to higher grades, whereas increasing strength of correlations as students get older is found between contextual problems and students' performance on the three large-scale tests.

**Keywords:** elementary school mathematics textbook, geometry, representation form, problem type, question format

## INTRODUCTION

Many studies have focused on the comparison of mathematics curriculum and mathematics textbooks (Baker, Knipe, Cummings, Blair, & Gamson, 2010; Cai, 2008; Cai & Ni, 2011; Fan, Zhu, & Miao, 2013; Reys, Reys, & Rubenstein, 2010; Schoen, Ziebarth, Hirsch, & BrckaLorenz, 2010; Usiskin & Willmore, 2008; Zhu & Fan, 2006). Baker et al. (2010) pointed out that mathematics textbooks can be regarded as the most accountable and important historical proof for the development of mathematics curriculum, research process, and the whole mathematics education history, which can help us realize the changes in a country's mathematics education. In addition, much research has shown that mathematics textbooks play a key role in the process of students' learning and teachers' teaching (Cai, 2008; Cai & Ni, 2011; Chavez, 2003; Fan et al., 2013; Gonzales et al., 2004; Reys et al., 2010; Stein, Remillard, & Smith, 2007). The quality of textbooks influences students' learning outcomes and mathematics achievement as well as teachers' teaching efficiency (Floden, 2002; Reys & Reys, 2006; Stein et al., 2007; Törnroos, 2004). This highlights the importance of mathematics textbooks in mathematics learning and teaching.

Geometry has been considered a key topic in school mathematics classes (Finnish National Board of Education, 2004; Ministry of Education in Taiwan, 2008; National Council of Teachers of Mathematics [NCTM], 2000). For example, NCTM (2000) claimed that geometry can help people depict the world in a systematic way. In Taiwan, the Ministry of Education (2008) strongly emphasizes the importance of geometry in school mathematics curriculum. In fact, geometry not only plays an important role in mathematics but also highly affects students' mathematics learning (Atiyah, 2001).

During the past two decades, many textbook studies have focused on algebra or fraction, but few textbook studies have focused on geometry currently, especially in international and comparative textbook studies. Due to the importance of textbook content, many mathematics educators claim that we can observe the advantages and disadvantages of textbooks from different countries by conducting textbook analyses, which can then be used to revise our textbooks in the future (Cai, 2008; Hiebert et al., 2003; Hong & Choi, 2014; Stigler & Hiebert, 2004). Based on the aforementioned motivations, the purposes of this study were to compare the differences in the use of geometry in elementary school mathematics textbooks among Finland, Mainland China, Singapore, Taiwan, and the USA and to investigate the relationships between the design of textbooks and students' performance on large-scale tests such as TIMSS-4 geometry, TIMSS-8 geometry, and PISA space and shape. The research questions are as follows:

1. Are there any differences in representation forms (symbolic form, verbal form, visual form, and combined form) of geometry among the five mathematics textbooks?
2. Are there any differences in problem types (contextual problem, non-contextual problem) of geometry among the five mathematics textbooks?
3. Are there any differences in question formats (open-ended question, close-ended question) of geometry among the five mathematics textbooks?
4. What are the relationships between the scores of TIMSS- 4 geometry, TIMSS-8 geometry, PISA space and shape and the frequencies of representation form, problem type, and question format?

## BACKGROUND

Previous studies have shown extreme differences in mathematics content and design used in textbooks from different countries (Author et al., 2010; Fan, 2013; Schmidt, 2004; Zhu & Fan, 2006). For example, although Asian countries such as Japan, Singapore, and Taiwan follow a national curriculum guideline, their textbooks still differ from each other. However, American textbooks do not have a national curriculum to follow (Schmidt, 2004). Reys, Reys, and Chavez (2004) discovered that American first grade mathematics textbooks usually consist of as many as 800 pages; on the contrary, Japanese and Taiwanese first grade mathematics textbooks include only one quarter of the pages of American counterparts, indicating that a large discrepancy exists in mathematics textbooks. According to the reports of the TIMSS and PISA tests, this kind of discrepancy can influence the performance in mathematics of students from different countries (Author et al., 2010; Fan, 2013; Schmidt, 2004; Zhu & Fan, 2006). Previous studies have pointed out that mathematics textbooks can affect students' opportunity to learn directly; in other words, the quality of textbooks can influence how students learn mathematics (Cai, 2008; Cai & Ni, 2011; Fan, 2013; Gonzales et al., 2004; Organization for Economic Co-operation and Development [OECD], 2013; Schmidt, 2004; Schmidt et al., 2001; Stein et al., 2007; Tarr, Chavez, Reys, & Reys, 2006).

Author et al. (2010) compared the differences in fraction for fifth and sixth grades among Taiwan's Kang Hsuan textbooks (KH), Singapore's My Pals Are Here Maths (MPHM), and America's Mathematics in Context (MiC). The results revealed two major differences in the textbooks from these three countries. The first was that over 90% of MiC consisted of contextual problems, whereas only 55% of Taiwan KH and 48% of Singapore MPHM were contextual problems. Another difference is that American MiC highly stresses conceptual knowledge (about 76%); however, Taiwan KH and Singapore MPHM have one-third of the problems focusing on the development of conceptual knowledge. A second difference is that Singapore MPHM placed fraction in fifth grade and stresses proportion in sixth grade. On the contrary, Taiwan KH and American MiC finished the whole fraction lessons in sixth grade.

Zhu and Fan (2006) compared problem representation in mathematics textbooks of Mainland China with their American counterparts. The results showed that American middle grade mathematics textbooks had high percentages of non-routine problems, non-traditional problems, open problems, application problems, and failing-to-meet-conditions problems. On the contrary, Mainland China had more multi-step problems, thus the percentage of these types of challenging problems was higher in textbooks from Mainland China. Stein et al. (2007) compared two American mathematics textbooks and found that presentational order, method, and organization of the textbook content were different from each other. Presentational order and organization of textbooks can influence students' learning opportunities.

Many studies pointed out that the textbook is one of the major factors that influences students' learning (Cai & Ni, 2011; Cai, Wang, Moyer, Wang, & Nie, 2011; Fan, 2013; Schmidt et al., 2005). Reys et al. (2004) further claimed that problem types and presentation of the materials in the mathematics textbook are important factors that affect mathematics teaching and learning. Many studies suggested that multiple representations should be appropriately integrated into the mathematics classroom to enhance students' conceptual understanding (Author et al., 2004; Cramer, Post, & delMas, 2002; NCTM, 2000; Rittle-Johnson & Koedinger, 2005; Sood & Jitendra, 2007). In addition, some studies suggested that visual forms should play an important role in mathematics teaching and learning (Bishop, 1991; Brenner, Herman, Ho, & Zimmer, 1999; NCTM, 2000; River, 2010; Zimmermann & Cunningham, 1991). Other studies pointed out that visual forms can help students construct geometrical concepts and facilitate students' visualizations of geometrical objects (Arcavi, 2003; David & Tomaz, 2012; Presmeg, 2006).

Additionally, some researchers have argued that mathematics learning should connect to real-world contexts. That is, real-world mathematics activities should be integrated into the classroom to enable students to better understand mathematics (Author, 2006; NCTM, 2000; Sood & Jitendra, 2007). Many studies found that increasing the number of real-world problems and applications in mathematics activities can help to diversify problems in mathematics textbooks. These kinds of mathematics activities can create a learning environment that helps students develop higher-level thinking and understanding (Author et al., 2010; Gu, Huang, & Marton, 2004; Griffin, 2004; Griffin & Jitendra, 2009; Van De Walle, 2007).

## METHOD

### Selection of Textbooks

Kang Hsuan (KH) elementary school mathematics textbooks have about a 38% market share in Taiwan (Kang Hsuan Educational Publishing Group, 2009) and are the most commonly used elementary school mathematics textbook series. There are eight or nine units for each textbook, a total of 115 units in the Kang Hsuan series for grades 1-6, among which 21 units deal with the topic of geometry.

Everyday Mathematics (EM), a set of 1st-6th grade mathematics textbooks, was created by the research institution of the University of Chicago based on the standards of NCTM (National Council of Teachers of Mathematics, 1989, 2000). EM is one of three sets of standards-based elementary mathematics textbooks and it is also the most representative elementary school textbook in the USA with the highest market share (15.9%) in the American elementary school textbook market (Reys & Reys, 2006). The Everyday Mathematics series includes 38 units, with 6 units devoted to geometry.

Laskutaito mathematics textbooks, a set of 1st-6th grade mathematics textbooks, were published in 2004 by Finland Werner Söderström Corporation (WSOY) (Rikala, Sintonen, Uus- Leponiemi, Ilmavirta, & Sieppe, 2006) and are based on the core mathematics curriculum development of Finland. Laskutaito has the highest proportion of the market share (70-80%) among Finland's elementary school mathematics textbooks (Chen, 2008). There are 54 units in Laskutai mathematics textbooks for grades 1-6, and 6 units are devoted to geometry.

My Pals Are Here! Maths (MPHM) in Singapore is a set of 1st-6th grade mathematics textbooks edited by Fong, Ramakrishnan, and Gan (2005) under an agreement with the Ministry of Education in Singapore (2001). The market share of MPHM is about 60% in elementary schools (Author et al., 2010). The MPHM series includes 93 units, 21 of which deal with the topic of geometry.

Mathematics textbooks in China were edited by elementary school mathematics educational experts, university professors, special teachers, and educational researchers. A set of textbooks were edited based on the current Chinese nine-year compulsory educational research. These kinds of textbooks have the highest proportion of the market share (40-50%) among Chinese elementary school mathematics textbooks (Ji, 2004; Ma &Tan, 2008). Such textbook series include 93 units for grades 1-6, with 18 units devoted to geometry.

### Analytical Framework

The problems and exercises in the student textbooks were counted to determine the total number of geometrical items. The following is an example item:

Example 1: Who sees the object from this viewpoint?

Since there are four blanks for filling in answers, this was counted as four problems.

To answer the first research question, representation forms used in the geometry problems in the student textbooks were classified as symbolic form, verbal form, visual form, and combined form based on the study of Zhu and Fan (2006). If a geometry problem included only mathematical expressions, then the problem was coded as symbolic form. For example, "Which of the following items can form a triangle? (1) 4 cm, 3 cm, 8 cm; (2) 3 cm, 3 cm, 3 cm; (3) 5 cm, 8 cm, 16 cm?" (From KH 5A, p. 48). If a problem was presented in verbal form only, then the problem was coded as verbal form. For example, "Jane draws a community plan with a ratio 1:2000. There is a square park in the community plan. The real length and width of this square park are 1 km and 0.8 km. What are the length and width of the square park of the community plan?" (From KH 6A, p. 128). If a problem was presented using figures, pictures, graphs, tables, and so on, then it was coded as visual form. For example, "Who can see the object from this viewpoint?" (see **Figure 1**). If two or more of the above representation forms were used in a problem, then it was coded as combined form. **Figure 2** is an example item:

 

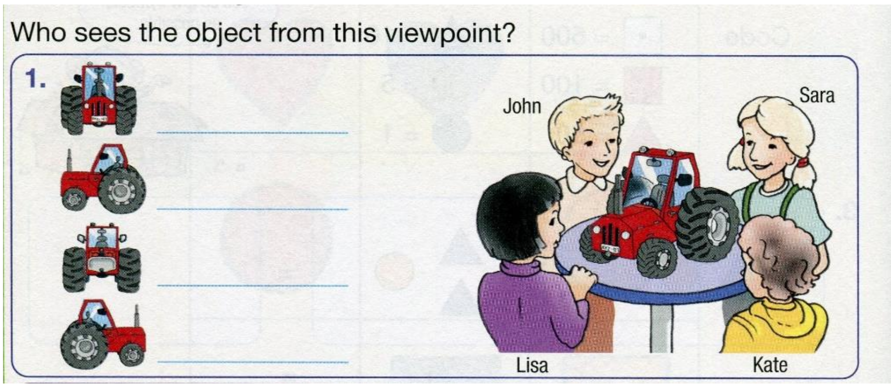


**Figure 1.** Example of counting problems and visual form from Laskutaito textbooks.
Note. From Laskutaito 2B, p. 44.

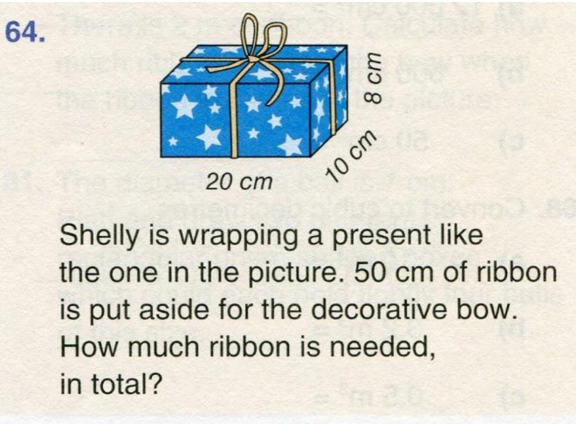


**Figure 2.** Example of combined form from Laskutaito textbooks.
Note. From Laskutaito 6A, p. 81.

To answer the second research question, problem types of the geometry problems in the student textbooks were classified as contextual and non-contextual problems (Author et al., 2010; Hiebert et al., 2003). If a problem was grounded in a real-world context, then it was coded as a contextual problem. If a problem was grounded in symbols in the language of mathematics, then it was coded as a non-contextual problem (Author et al., 2010; Hiebert et al., 2003). For example, "Which of the following items can form a triangle? (1) 4 cm, 3 cm, 8 cm; (2) 3 cm, 3 cm, 3 cm; (3) 5 cm, 8 cm, 16 cm?" (From KH 5A, p. 48).

To answer the third research question, question formats of the geometry questions in the student textbooks were classified as open-ended and close-ended questions (Zhu & Fan, 2006). An open-ended question is defined as a question with many correct answers (Zhu & Fan, 2006). A close-ended question is defined as a question with only one correct answer (Zhu & Fan, 2006).

To answer the fourth research question, the scores of TIMSS- 4 geometry, TIMSS-8 geometry, PISA space and shape from the reports of TIMSS 2011 test (Mullis, Martin, Foy, & Arora, 2012) and PISA 2012 test (OECD, 2013) and the frequencies of representation form, problem type, and question format from elementary school mathematic textbooks were used in the present study.

**Inter-Rater Reliability**

In the present study, one mathematics educator and two graduate students served as raters for coding data. Based upon the coding framework, the raters independently coded the problems in the student textbooks. The inter-rater reliability coefficient for the raters was found to be 0.94.

 

## RESULTS

### Differences in representation forms of geometry among the five mathematics textbooks

The chi-square test for independence was used to evaluate whether representation form was related to country. In Table 1, the chi-square test showed that there was a significant association between representation form and country ( (df = 12) = 400.15, $p < 0.001$). This means that the percentages of representation forms in the textbooks from the five countries are significantly different from one another. A comparison of the representation forms and the four sets of percentages for each of the five countries shows that Mainland China had the highest percentage of symbolic form ($z = 5.9$) and highest percentage of verbal form among all countries; Finland ($z = 5.8$) had the highest percentage of visual form, and Singapore ($z = 11.2$) had the highest percentage of combined form.

**Table 1.** Cross-tabulation of representation form and country with adjusted residuals

| Representation form | Country | | | | | |
|---|---|---|---|---|---|---|
| | Finland | Mainland | Singapore | Taiwan | USA | Total |
| Symbolic form | 13 | 31 | 59 | 3 | 1 | 107 |
| (Percentage) | (12.1%) | (29.0%) | (55.1%) | (2.8%) | (0.9%) | (100%) |
| (Adjusted residual) | (-3.3) | (5.9) | (5.1) | (-4.5) | (-3.2) | |
| Verbal form | 162 | 171 | 155 | 211 | 131 | 830 |
| (Percentage) | (19.5%) | (20.6%) | (18.7%) | (25.4%) | (15.8%) | (100%) |
| (Adjusted residual) | (-4.9) | (9.6) | (-9.4) | (4.4) | (5.7) | |
| Visual form | 573 | 206 | 536 | 407 | 157 | 1879 |
| (Percentage) | (30.5%) | (11.0%) | (28.5%) | (21.7%) | (8.4%) | (100%) |
| (Adjusted residual) | (5.8) | (-0.4) | (-4.7) | (2.4) | (-3.9) | |
| Combined form | 351 | 62 | 606 | 219 | 147 | 1385 |
| (Percentage) | (25.3%) | (4.5%) | (43.8%) | (15.8%) | (10.6%) | (100%) |
| (Adjusted residual) | (-0.8) | (-9.7) | (11.2) | (-4.8) | (0.4) | |
| Total | 1099 | 470 | 1356 | 840 | 436 | 4201 |
| (Percentage) | (26.2%) | (11.2%) | (32.3%) | (20.0%) | (10.4%) | (100%) |

### Differences in problem types of geometry among the five mathematics textbooks

The chi-square test of independence was used to explore whether there is a relationship between problem type and country. The Pearson chi-square value is statistically significant, (df = 4) = 405.72, $p < 0.001$. This means that the percentage for the two problem types varied as a function of country. Looking at Table 2, it can be seen that Mainland China and Taiwan had the higher percentages of contextual problems ($z = 16.9$ and $z = 5.4$, respectively); on the contrary, Singapore and the USA had higher percentages of non-contextual problems ($z = 12.5$ and $z = 5.1$, respectively).

**Table 2.** Cross-tabulation of problem type and country with adjusted residuals

| Problem type | Country | | | | | |
|---|---|---|---|---|---|---|
| | Finland | Mainland | Singapore | Taiwan | USA | Total |
| Contextual problem | 167 | 197 | 72 | 181 | 31 | 648 |
| (Percentage) | (25.8%) | (30.4%) | (11.1%) | (27.9%) | (4.8%) | (100%) |
| (Adjusted residual) | (-0.2) | (16.9) | (-12.5) | (5.4) | (-5.1) | |
| Non-contextual problem | 932 | 273 | 1284 | 667 | 405 | 3561 |
| (Percentage) | (26.2%) | (7.7%) | (36.1%) | (18.7%) | (11.4%) | (100%) |
| (Adjusted residual) | (0.2) | (-16.9) | (12.5) | (-5.4) | (5.1) | |
| Total | 1099 | 470 | 1356 | 848 | 436 | 4209 |
| (Percentage) | (26.1%) | (11.2%) | (32.3%) | (20.1%) | (10.4%) | (100%) |

### Differences in question formats of geometry among the five mathematics textbooks

The chi-square test for independence was used to determine whether there is an association between question format and country. A chi-square test for independence indicated a significant association between question format and country ( (df = 4) = 630.32, $p < 0.001$). This seems to indicate that the percentage of open-ended questions is significantly different from the percentage of close-ended questions for the textbooks from the five countries. Table 3 shows that Mainland China and the USA had the higher percentages of open-ended questions ($z = 16.3$

and z = 15.7, respectively), and conversely, Finland and Singapore had the higher percentages of close-ended questions (z = 10.1 and z = 11.8, respectively).

**Table 3.** Cross-tabulation of question format and country with adjusted residuals

| Question format | Country | | | | | |
|---|---|---|---|---|---|---|
| | **Finland** | **Mainland** | **Singapore** | **Taiwan** | **USA** | **Total** |
| Open-ended question | 54 | 150 | 29 | 98 | 140 | 471 |
| (Percentage) | (11.5%) | (31.8%) | (6.2%) | (20.8%) | (29.7%) | (100%) |
| (Adjusted residual) | (-10.1) | (16.3) | (-11.8) | (1.3) | (15.7) | |
| Close-ended question | 1408 | 320 | 1327 | 750 | 296 | 4101 |
| (Percentage) | (34.3%) | (7.8%) | (32.4%) | (18.3%) | (7.2%) | (100%) |
| (Adjusted residual) | (10.1) | (-16.3) | (11.8) | (-1.3) | (-15.7) | |
| Total | 1462 | 470 | 1356 | 848 | 436 | 4572 |
| (Percentage) | (32.0%) | (10.3%) | (29.7%) | (18.5%) | (9.5%) | (100%) |

**The relationships between the scores of TIMSS-4 geometry, TIMSS-8 geometry, PISA space and shape and the frequencies of representation form, problem type, and question format**

Correlation analysis was performed to explore the relationships between some pairs of variables (e.g., the scores of TIMSS-4 and 8 geometry and PISA space and shape and the frequencies of verbal form, visual form, combined form, contextual problem, and open-ended question). As can be seen from **Table 4**, there was a strong positive relationship between visual form and TIMSS grade 4 geometry (r = 0.502), followed by a moderate positive relationship between visual form and TIMSS grade 8 geometry (r = 0.386) and a weak positive relationship between visual form and PISA age 15 space and shape (r = 0.086). The results indicated that the strength of the positive relationships between visual form, TIMSS-4 geometry, TIMSS-8 geometry, and PISA space and shape decreases as students advance to higher grades. Likewise, the strength of the positive relationships between combined form, TIMSS-4 geometry, TIMSS-8 geometry, and PISA space and shape followed a similar pattern, indicating that the values of correlation coefficients decrease as students move from grade 4 (r = 0.702, large) to grade 8 ( r = 0.432, medium) to age 15 (r = 0.100, small). Conversely, the strength of the positive relationships between contextual problem, TIMSS-4 geometry, TIMSS-8 geometry, and PISA space and shape reversed the pattern, that is, the values of correlation coefficients increase as students get older from grade 4 (r = 0.149, small) to grade 8 ( r = 0.349, medium) to age 15 (r = 0.630, large). Additionally, there were two negative correlations and one positive correlation between open-ended question, TIMSS-4 geometry, TIMSS-8 geometry, and PISA space and shape. Specifically, the results showed that there was a strong negative relationship between open-ended question and TIMSS-4 geometry (r = -0.619), with high frequency of open-ended question associated with a low score on TIMSS-4 geometry; a moderate negative relationship between open-ended question and TIMSS-8 geometry (r = -0.408), with high frequency of open-ended question associated with a low score on TIMSS-8 geometry; and a weak positive relationship between open-ended question and PISA space and shape (r = 0.004), with high frequency of open-ended question associated with a high score on PISA space and shape. Finally, the results showed that there were three positive correlations between verbal form, TIMSS-4 geometry, TIMSS-8 geometry, and PISA space and shape; however, there was no pattern evident in the relative strength of the correlations. In other words, there was a moderate positive relationship between verbal form and TIMSS-4 geometry (r = 0.474), a strong positive relationship between verbal form and TIMSS-8 geometry (r = 0.720), and a strong positive relationship between verbal form and PISA space and shape (r = 0.502).

**Table 4.** Pearson correlation coefficients (*r*) between each pair of variables listed

| Test | Representation form/ Problem type/Question format | | | | |
|---|---|---|---|---|---|
| | **Verbal form** | **Visual form** | **Combined form** | **Contextual problem** | **Open-ended question** |
| TIMSS grade 4 geometry | 0.474 | 0.502 | 0.702 | 0.149 | -0.619 |
| TIMSS grade 8 geometry | 0.720 | 0.386 | 0.432 | 0.345 | -0.408 |
| PISA age 15 space and shape | 0.502 | 0.086 | 0.100 | 0.630 | 0.004 |

 

## DISCUSSION

The results showed that there are significant differences in the representation forms among the five elementary school mathematics textbooks at a 0.001 level of significance. In particular, the Singapore textbooks have the highest percentage of the combined form. It appears that Singapore elementary school mathematics textbook series (MPHM) highly focuses on the visual form combined with other representation forms. Previous studies suggested that mathematics learning should put more emphasis on choosing different representations to appropriately help students make sense of the mathematics and further be able to have transformation ability among multiple representations, which can help students understand more deeply about mathematics content (Author et al., 2004; Cramer et al., 2002; NCTM, 2000; Rittle-Johnson & Koedinger, 2005; Sood & Jitendra, 2007). The high usage of a combined form including visual form and other different representation forms in Singapore elementary school mathematics textbooks may be the key factor that caused students from Singapore to perform well on international assessments.

The results also showed that there are significant differences between contextual and non-contextual problems among the five elementary school mathematics textbook series at a 0.001 level of significance. In particular, Mainland China has the highest percentage of contextual problems. Previous studies suggested that contextual problems in mathematics textbooks can promote students' mathematics learning, enhance students' conceptual understanding in mathematics (Author, 2006; Author et al., 2010; Griffin, 2004; Julie, 2013; Van De Walle, 2007; Zhu & Fan, 2006), and create a learning environment to develop higher-order mathematical thinking (Gu, Huang, & Marton, 2004). The new mathematics textbooks based on the ideas of reform of mathematics education in Mainland China seem to reflect the requirements of the international mathematics community, which encourages that contextual problems be integrated into mathematics teaching and assessment to improve students' performance in mathematics (Julie, 2013; NCTM, 2000; OECD, 2013; Zhu & Fan, 2006). Mathematics textbook developers from Singapore and the USA may consider integrating more contextual problems related to geometry into mathematics textbooks.

The mathematics textbooks from Mainland China and the USA seem to include more open-ended geometrical questions. Previous studies suggested that open-ended questions provide students with more opportunities to solve higher-order thinking and challenging problems than closed-ended questions (Cai, 1995; Cai & Ni, 2011; Zhu & Fan, 2006). The study by Kwon, Park, and Park (2006) showed that the use of open-ended questions in mathematics classrooms can cultivate students' divergent thinking skills, including fluency, flexibility, and originality. Mathematics textbook developers from Finland and Singapore may consider including more open-ended questions related to geometry in mathematics textbooks.

The results of the correlation analysis showed that the strength of the positive relationships between visual form (combined form), TIMSS-4 geometry, TIIMSS-8 geometry, and PISA space and shape decreases as students advance to higher grades. Findings suggest that younger students may benefit more from a visual form and combined form. These findings support the findings of previous studies, indicating that the design of mathematics textbooks can affect students' learning outcomes (Cai, 2008; Cai & Ni, 2011; Floden, 2002; Gonzalez et al., 2004; Schmidt, 2004; Schmidt et al., 2001; Stein, Remillard, & Smith., 2007; Tarr et al., 2006; Törnroos, 2004). Since the combined form contains mostly a visual form, it is natural to conjecture that the visual form is the major one related to students' performance in mathematics.

The strength of the positive relationships between contextual problem, TIMSS-4 geometry, TIIMSS-8 geometry, and PISA space and shape increases as students get older, indicating that the contextual problems concerning geometry in elementary school mathematics textbooks may be positively associated with students' future learning in reading and problem-solving abilities. One possible explanation related to this association could be that a strong positive relationship between contextual problems and PISA space and shape is due to the PISA test questions reflecting different aspects of the real world, which is similar to contextual problems in mathematics textbooks. Mainland China has the highest percentage of contextual problems in elementary school mathematics textbooks, which may explain Chinese students' best performance on PISA 2009 and 2012 tests (OECD, 2010, 2013). The high frequency of open-ended questions associated with low scores on TIMSS-4 geometry and TIMSS-8 geometry may be attributable to the multiple-choice format of test questions on those two assessments. However, more studies with a larger sample size may be needed to confirm these findings.

## CONCLUDING REMARKS

The present study makes three major contributions to the topic of geometry in elementary school mathematics textbooks. First, few studies have examined the differences in elementary school mathematics textbooks on the topic of geometry. The present study reports the differences in representation forms, problem types, and question formats among mathematics textbooks from five countries and the relationships between the scores of TIMSS- 4 geometry, TIMSS-8 geometry, and PISA space and shape and the frequencies of representation form, problem type, and question format of mathematics textbooks from those five countries. These differences can serve as a benchmark for future textbook design or revision on geometry for different countries, especially as students from Finland, Mainland China, Singapore, and Taiwan perform at the top in mathematics on the TIMSS and PISA tests (Mullis, Martin, Gonzalez, & Chrostowski, 2004; Mullis, Martin, Foy, & Arora, 2012; OECD, 2010, 2013). Second, the results showed that there is a strong positive relationship between visual form and TIMSS-4 geometry, a moderate relationship between visual form and TIMSS-8 geometry, and a weak relationship between visual form and PISA space and shape. Findings suggest that visual form may play an important role in mathematics textbooks. These findings support the findings of previous studies that visual form can help students construct geometrical concepts and facilitate students' visualizations of geometrical objects (Arcavi, 2003; David & Tomaz, 2012; Presmeg, 2006) and highlight the importance of visual form in mathematics teaching and learning (Bishop, 1991; Brenner, Herman, Ho, & Zimmer, 1999; NCTM, 2000; River, 2010; Zimmermann & Cunningham, 1991). Third, the trend of associations between the combined form and the three large-scale tests also decreases as students get older. This seems to highlight the importance of multiple representations in elementary school mathematics textbooks. These findings support the findings of previous studies, indicating that multiple representations may play a crucial role in mathematics education (Author et al., 2004; Cramer, Post, & delMas, 2002; NCTM, 2000; Rittle-Johnson & Koedinger, 2005; Sood & Jitendra, 2007).

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# A Balanced Approach to Building STEM College and Career Readiness in High School: Combining STEM Intervention and Enrichment Programs

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**ABSTRACT**

Often STEM schools and STEM enrichment programs attract primarily high achieving students or those with strong motivation or interest. However, to ensure that more students pursue interest in STEM, steps must be taken to provide access for all students. For a balanced and integrated career development focus, schools must provide learning opportunities that are appropriate for all students. This paper outlines two approaches to the creation of a comprehensive STEM College and Career development pathway in high schools.

**Keywords:** high school STEM program development, STEM education, STEM college and career development

## INTRODUCTION

TEM workers currently make up nearly 20% of the workforce in the U.S. and are projected to expand further in the next decade (Xue & Larson, 2015); however, whether or not the U.S. can produce enough STEM professionals to meet future demand is the more important question. The President's Council of Advisors on Science and Technology projected that the country will need one million more STEM workers in the next decade to remain globally competitive in science and technology (PCAST, 2012). However, the Census Bureau predicts that the number of STEM undergraduates will decline by a million students by 2025 (NSB, 2014).

One potential contributing factor cited in the current STEM gap debate is the low number of diverse and rigorous STEM courses in American high schools (USDOEOCR, 2014). According to the data from U.S. Department of Education (DOE) Civil Rights Data Collection only 50% and 63% high schools nationwide offer calculus and physics, respectively (2014, page 1). Furthermore, about 10-25% of high schools in the country offer no more than one STEM core courses (USDOEOCR, 2014, page 1). These statistics highlight the importance of building a STEM college and career pathway for high school students, which will lead to increased interest and participation in STEM.

Another concern often discussed among educators is the limited or lack of connections between STEM learning experiences inside and outside of the classroom. The U.S. government has spent $2.8 billion to 3.4 billion funding STEM education annually (Gonzalez & Kuenzi, 2012), much of which is allocated to intervention and enrichment programs that are outside of the school day. Since many of these programs operate apart from the school, many educators are often left in the dark about program outcomes, content, and how to create extension learning opportunities within the classroom. Many schools continue to struggle to find the best approach to building a comprehensive STEM college and career pathway to meet the needs of all high school students. The purpose of this article is to illustrate an integrated approach to designing an inclusive STEM college and career pathway.

**Approaches to High School STEM College and Career Readiness**

Creating a pathway to STEM college and career readiness in high schools includes both access to rigorous college and career preparatory coursework (i.e. Advanced Placement courses, STEM elective classes) and opportunities to extend STEM learning during out-of-school time (i.e. afterschool STEM clubs, fieldtrips, summer programs, afterschool tutoring/mentoring). Approaches to development can be either short-term or long-term depending on program goals, school resources, and student needs. Common short-term approaches may include single-day events (fieldtrip, guest speaker) or short-duration programs (weekend workshop series). Examples of long-term strategies may include a sequence of courses or activities that provide continuous STEM learning opportunities distributed over longer periods of time. The frequency and duration of these programs can vary and can be utilized during both in-school and out-of-school time (Wilkerson & Haden, 2014). It is important to keep in mind that emphasis on any one approach will often fall short of meeting the college and career development needs of all students.

The first step for educators is to critically reflect on the strategies and approaches to STEM college and career development they utilize and identify opportunities outside of the school that can be incorporated during school to further enhance student development (Buckner & Boyd, 2015). Creating a comprehensive program to address student needs requires careful planning which is guided by a thorough student-driven needs assessment, includes the voices of all stakeholders, and leverages all resources and funding strategically. As in the design of any academic program, specific objectives and outcomes measures must be the driving force in the program development and evaluation. If planned and implemented adequately, these programs produce not just short-term, but also long-term positive effects, which include increased student engagement and achievement in coursework, enrollment in a higher education STEM programs, and interest in pursuing STEM careers (Markowitz, 2014).

**Enrichment and Intervention: Goals of High School STEM Education Programs**

High school supplemental STEM programs generally fall within two categories: Enrichment or Intervention. Enrichment programs typically attract high-achieving and highly motivated students, while intervention programs target specific groups of students such as women, ethnic minorities, low achieving students, etc., in hope of elevating motivation and skills. Both types of programs aim to increase exposure and engagement in STEM learning, and both are necessary components in a comprehensive approach to developing students' college and career readiness in STEM. The goals for high school STEM programs should be twofold: first develop students' interest in STEM fields, then bridge the gap between interest, skill, and knowledge to provide students the requisite preparation for STEM majors and careers (Valla & Williams, 2012).

**Enrichment Programs.** STEM enrichment programs or activities are specifically designed to enhance core instruction and provide addition extension opportunities for application of learning. For example, schools who offer an elective robotics course will also offer an after school robotics club where students are given the opportunity to work on class projects and incorporate new learning that may not be covered in the course curricula. These programs are essential to address the growing concern over the inability to cover all course concepts in depth due to time constraints. Another example, created by North Carolina School of Science and Mathematics, is a distance education and extension program that includes live sessions with interactive opportunities to provide additional hands-on activities and online "do-it-yourself" enrichment lessons that offer materials, videos, and guidance for independent study (http://fluorine.ncssm.edu/learn/stem_enrichments).

**Intervention Programs.** Without STEM intervention programs many underrepresented populations may have lacking or negative STEM experiences, which may lead to misconceptions about STEM careers, disinterest, or low perceptions of ability (Valla & Williams, 2012). Although, some of these programs also have an enrichment focus, the primary emphasis is fostering equity in experience and opportunity for a population where there is current or historic evidence of disparity in access, achievement, or participation in STEM fields. One example is offing a guest speaker series for students with disabilities where the speakers are individuals with disabilities who work in STEM fields and discuss both aspects of their job and overcoming barriers to attaining their career goals. Examples of a few popular after school or summer intervention programs include: Girls Who Code, For Inspiration and Recognition of Science and Technology (FIRST), Boosting Engineering Science and Technology (BEST), Advanced Technological Education (ATE), Operation SMART, AHC Mentor Program, Build IT, Digital Wave, VEX.

**A Comprehensive STEM College and Career Readiness Program**

A comprehensive STEM college and career readiness program can be achieved when schools offer both enrichment and intervention programs that combine short-term/long-term and in-school/out-of-school learning opportunities. Research on supplemental after school STEM programs indicate a correlation between program participation and an increase in pursuit of a higher education study in a STEM field (After School Alliance Report, 2011). Other positive effects of both short-term and long-term STEM programs are improved attitude in STEM fields, increased STEM knowledge and skills, higher chance of graduation and pursuing STEM careers (Afterschool Alliance, 2011). Combining both after school and during school STEM programs and activities that provide students with intervention and enrichment opportunities are key to developing a STEM College and Career Pathway that ensures readiness for all students.

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