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Descriptive Analysis of the Graphic Representations of Science Textbooks

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ABSTRACT

Textbooks are primary teaching aids, sources from which students obtain knowledge of science domain. Due to this fact, curriculum developers in the field emphasize the crucial role of analysing the contents of science textbooks in improving science education. Scientific domain knowledge relies on graphical representations for the manifestation of itself. Content analysis of the representations therefore entails the necessity of the systematic reading and categorizing of a body of visual representations, including diagrams, drawings, photos, and text that appeared in these science teaching and learning materials. This paper examined ten UAE primary science textbooks and practical books using an author-improved graphical analysis grid. From the descriptive analysis, it was found that general science was the illustrated topic area for fundamental level of science. The most frequently used graphical type is the iconic. Female images outnumber the male images. Results also showed that indigenous graphics were dominant over foreign graphics. The study also found the majority of visual representations in the textbooks were designed to convey static information and they are in close relationship with the textual representations. Yet most of the graphical images were indexed and captioned that served specific cognitive functions. The results also suggest graphic representations need to be treated as an important visual tool that textbook authors should use them wisely to have the domain knowledge conveyed in various topic areas.

Keywords: graphic representations, coding scheme, content analysis

INTRODUCTION

Textbooks play a crucial role in the science teaching and learning. As all science subjects seek to provide representations and explanations for natural phenomena so as to describe the causal relationships and the complexity of the natural world (Gilbert, 2007). For this reason, science educators consider textbooks as instructional resources that support teachers in planning and delivering science instruction to meet local and national curricular standards. Textbook authors thus enjoyed the freedom to manifest the national curricula and examination specifications (Martinez-Gracia, Gil-Quilez and Osada, 2006). As a major source of information in teaching and learning of a particular science subject, the quality and accuracy of the textbook content is crucial for their educational effectiveness.

Textbook researchers aimed at exploring different aspects of textbook features. The research territories may include the treatment of socio-scientific and controversial issues, analysis of representational information, portrayal and convey of knowledge (Orgill, 2013), nature of science (e.g., Abd-El-Khalick et al., 2017), gender issues (e.g., Yasin et al., 2012), comprehensibility of text (e.g., Kloser, 2016), language pattern (e.g., Muspratt and Freebody, 2013), demonstration of indigenous knowledge (Rillero, 2013), and cultural and religious sensibility (e.g., Wiseman, 2014). Various forms of analytical tools have been developed: questionnaires, rubrics, grids, criteria, rating scheme, protocols, and coding of the indicators.

Criteria were formulated for the analysis of texts and various modes of presentations, such as illustrations, photos, maps, tables and exercises. A guidebook proposed by the UNESCO provided practical advice for textbook reviewers in adopting both quantitative and qualitative methods (Pingel, 2010). It suggested examining the proportion of facts and views and examiners' interpretation. The American Association for the Advancement of Science, (AAAS) Project 2061 developed protocols evaluating the instructional effectiveness of science textbooks. Science educators observed when students learn science topics; there is a need to emphasize deep conceptual understanding rather than factual memorization. As such, a wide repertoire of content-specific instructional supports that promote understanding among students from diverse backgrounds, interests, and abilities (Koppal and Caldwell, 2004).

Existing literature recognized the the role of graphical representations in constructivist learning, which is not only to transmit information but also to enable students conduct their own investigation. Studies explained the role of graphical displays in learning and thus effective graphical designing principles should be synthesized. By examination of three theoretical perspectives on visual learning, Vekiri (2002) concluded that dual *coding theory*, *visual argument*, and *conjoint retention* are compatible although each theory has its own assumptions. Vekiri also claimed that graphics are effective learning tools only when they allow learners to interpret and integrate information with minimum cognitive processing. Learner's differentiation such as prior subject knowledge, visuospatial ability, and learning strategies could influence graphic processing and interact with graphical design to mediate its effects.

In particular, studies on engaging visual representations in science books could be roughly divided into two perspectives (Liu and Khine, 2016). The first approach mainly focuses on the instructional effectives of visual displays brought to the individual learning. A majority of studies followed this vein and examined the leaners' visual learning. Slough et al. (2010) quantified the type and quality of the graphical representations in middle school textbooks in the United States and examined how they interacted with the texts. An instrument named Graphical Analysis Protocol (GAP) designed by Slough's research team to analyse the graphic types the interaction with the textual material. In their findings, they reported most graphics were static, and analytic forms of graphics serve decorative purpose in nature. Moreover, they found about one third of the graphs were not spatially or semantically related to the text. Irez (2010) examined five secondary school biology textbooks and examined the treatment of nature of science. Lee (2010) examined the design and use of representations changed over the six decades in the USA, and argued graphs used in the textbooks become increasingly iconic. It was found science was portrayed as a collection of facts rather than of a dynamic process of generating and testing alternative explanations about the nature of science. The study identified one problem in the textbooks that potentially generate alternative conceptions about science as the scientific domains was portrayed as congregation of facts rather than dynamic processes.

The second approach considers the school books as cultural objects that may have an impact on the representational teaching and learning activities. Since textbooks represent the national curricular and the uniqueness of cultural identity. Researchers of this orientation believed that textbooks are developed to carry the national perspectives and culture (Izquierdo and Gouvea, 2008). There are also science educators insist on incorporating indigenous knowledge and local cultural of the society if to make textbooks effective in their own contexts (Ninnes, 2000). As for gender issues, Kahveci (2010) conducted a quantitative study to investigate gender equity, questioning level, science vocabulary load, and readability level. The author concluded that the textbooks included unfair gender representations, a considerably higher number of input-level and processing than output level questions, and high load of science terminology.

Reading visual displays for the scientific understanding could be a challenging task. More recent attention has focused on the analysis of specific textbook designing attributes when different visual modes of representations are engaged. Previous research into the content analysis of diagrams usage in science textbooks focused on the designing features by examining the relations between images and teachers' instructional practice (Pozzer and Roth, 2003). In a longitudinal study by Lee (2010), the author examined the extent to which changes in representations in textbooks published in the US over the past six decades. It was found that high-fidelity images, such as photographs are more likely to be used than the schematic and explanatory images to promote the familiarization to students. In explaining the criteria for evaluating the quality of science textbooks, Devetak and Vogrinc (2013) noted that visual materials are sometime used to stimulate recall of prior information. Although many variables regarding graphic features were taken into consideration, the researchers failed to extract the overall graphical/representational of primary science textbooks.

Later, textbook studies tend to build connections between visual mode of representations with other variables, such as students' achievement or teachers' instruction. Liu and Treagust (2013) conducted a content analysis on diagrams used in the Western Australia science textbooks. They explored the distribution patterns of different types of diagrams in secondary science and biology textbooks and the uniformity of the frequency of usage.

Through observing teachers' classroom instruction, Liu and Treagust (2013) proposed a series of teaching practices facilitating science teaching when diagrammatic representations are used. More recently, Liu and Khine (2016) performed a descriptive statistical analysis on the distribution of diagrams contained in Bahrain primary science textbooks and workbooks. They tested statistical difference between different book categories, suggestions were also provided for teachers to improve the diagrammatic teaching efficacy. In another empirical study, Renkl and Scheiter (2017) identified the challenges learners may encounter when dealing with visual displays. They further elaborated the supportive interventions and evidence about learning outcomes. All the above studies unanimously emphasized the role of different means of instructional interventions (textbooks or teachers' teaching) in either reducing the visual displays' complexity or increasing the individuals' prerequisites for an effective visual learning.

Previous studies failed to specify the functionalities of different graphical types used in the textbooks. In identifying different types of graphs, a coding scheme was created according to the taxonomy proposed by Hegarty, Carpenter, and Just (2016) that classified graphics used in science teaching context into three types, which include: Iconic diagrams, schematic diagrams, and charts and graphs. Iconic diagrams are realistic pictures or drawings of concrete objects. They are effective in helping student recognize the appearance and structure that are available to visual inspection. An example of iconic diagram could be a drawing or a photo of an insect. The iconic sketches provide visible outlines that could help to identify the shape of the insect. Compared to iconic diagrams, schematic diagrams are rather abstract representations, such as phylogenic trees or electric circuit. Schematic diagrams are highly abstracted from the real-world entities but only preserve the physical relationships of the target information. Consequently, interpretation of a schematic diagram requires learners to decipher the abstract content of the diagram and make a connection to the target concept. The third category, charts and graphs, presents the relationships of quantitative data. For example, a line graph can display the change in the human population over the years, and a pie chart can show the percentage of carbon dioxide in air. It is often necessary for the reader to identify all independent variables before making an interpretation because abstract meanings and numerical data embedded into charts and graphs.

The study also includes Augmented Reality (AR) diagrams into the coding scheme, because the textbooks contain virtual reality images that were designed and produced by multimedia printing technology. AR diagrams can be thought of as the "middle ground" between completely synthetic and completely real. The information conveyed through the augmented reality images could help perform real world tasks (Renkl and Scheiter, 2017). All diagrams in the ten school books were analysed according to the four diagrammatic types – iconic, schematic, charts and graphs, and augmented reality.

RESEARCH OBJECTIVES

The overwhelming majority of scientific concepts rely on the illustration of diagrams for effective teaching and learning. Although visual displays have great potential to foster learning, they also pose substantial demands on learners. As the literature calls for more research to examine the information on textbook alignment and to work out new content analysis standards (Hegarty et al., 1991). Meanwhile, under the belief that diagrams were not randomly used by textbook authors, researchers of this study aim to explore the inherent rules that graphics by which textbook writers and science educators use them for the purpose of facilitating students' conceptual learning. Well-designed science learning materials are believed to have the potential in facilitating students' conceptual learning. Through analysing the diagrammatical usage in textbooks, this study could set an example and to provide methodological options for future textbook studies. The researchers of this study sought to explore a number of properties inherent in the graphic displays in UAE primary science books. The investigation of graphic representations in UAE context may provide more suggestions and experiences on the instructional usage for further book studies. As been previously mentioned, textbook quality has been correlated directly and indirectly to the success of educational reform and enhancement of students' understanding (Azuma, 1997). Curriculum of the primary science is the starting point for higher level of science learning. Undoubtedly, the learning at the higher level of science demands a gradual build-up of the previous learning. The critical analysis of graphical representations in primary science textbooks serves as an important strategy to reflect on how the designing features of representations are aligned with school science teaching and learning.

Research Questions

The research aim of the study was clarified as to centre on the overall usage of graphics across textbooks and practical books, rather than gain a profound understanding of a certain scientific topic explained by certain representational mode(s). The aim of this study was to explore the distributions and frequencies of graphic representations in the primary science schoolbooks. In addition, the original coding scheme was applied in a new context. Therefore, a number of research questions were formulated in related to the categories contained in the coding scheme:

(1) What are the frequencies of the graphics among the science topic areas?
(2) How various forms of graphical representations distributed in textbooks and practical books?
(3) What is the distribution of the gender representations of graphics in the sampled textbooks and practical books?
(4) How was the ethnic information distributed in the textbooks and practical books?
(5) How do the graphic representations relate to the text? The frequency of indexing relation between a diagram and text.
(6) How do the graphic representations relate to the text? The frequency of captioning relation of illustrations.
(7) What are the functional connections between the graphics and the written text (*decoration or relation*)?
(8) What was the quality feature (dynamic or static) of the graphical representations in the textbooks and practical books?

THEORETICAL FOUNDATION

Conceptual change perspectives of teaching and learning have provided a powerful framework for the research in science education as well as instructional design (Polikoff, Zhou and Campbell, 2015). Over the past decades, cognitive development approaches to conceptual change have undergone a shift from Piagetian development psychology that emphasizes stage-dependent and domain-general conceptual learning to other paradigms such as Ausubel's (1968) assimilation, Vygotskian perspectives (Strike and Posner, 1982). Ausubel (Cheng and Gilbert, 2009; Eilam and Poyas, 2010) believes the most important factor influences learning is what the learners already knows and hence to teach accordingly. Piaget's (Chamblisss and Calfree, 1989)argument emphasizes the interplay of assimilation and accommodation in classifying students' conceptions on explications of their thoughts and science concepts.

It is therefore evident to note conceptual change theory emphasizes the crucial roles of active engagement and students' existing knowing play in the individual learning. Textbook as a ubiquitous instructional material used in science teaching and learning, the effectiveness of students' learning process could be supported by addressing the defining features of students' individual's active engagement and the diagrammatic distribution in the primary science curriculum. Learning science with diagrams is grounded in the primary curriculum. Learning science with graphics is grounded in the conceptualisation of knowledge as a tentative human construction widely known as constructivism that insists conceptualisation is reflected in constructing the new knowledge on a prior conceptual scaffold. In this study, all visual representations were considered as constructive learning tools that could facilitate leaners' conceptual changing process.

RESEARCH DESIGN

Sample

The sample consisted of all Science Students' Textbooks and Practical Activity Books adopted in all government schools in the UAE that typically cover 5 years of primary instruction. Primary textbooks were selected as the sample in this study was due to the importance of preliminary level of schooling is the crucial stage for science learning. All these ten books were selected and analysed in the study are currently used in the UAE primary schools. **Table 1** lists the titles that were covered in this study.

Table 1. List of textbooks under study

No	Level	Title
1	Primary Grade 1	Science Around You 1 (Student Book)
2	Primary Grade 2	Science Around You 2 (Student Book)
3	Primary Grade 3	Science Around You 3 (Student Book)
4	Primary Grade 4	Science Around You 4 (Student Book)
5	Primary Grade 5	Science Around You 5 (Student Book)
6	Primary Grade 1	Science Around You 1 (Practical Activity Book)
7	Primary Grade 2	Science Around You 2 (Practical Activity Book)
8	Primary Grade 3	Science Around You 3 (Practical Activity Book)
9	Primary Grade 4	Science Around You 4 (Practical Activity Book)
10	Primary Grade 5	Science Around You 5 (Practical Activity Book)

Method

Visual analysis is a systematic, observational method that allows the quantification of samples of observable content classified into distinct categories (Slough et al., 2010). By using frequency test, the authors of this study conducted a quantitative content analysis that consists of three research phases including: 1) defining the modes of representation to be analysed， 2) compiling the coding scheme to be used to code the graphics, and 3) encoding the graphics and to gain inter-coder reliability. According to the research framework by Bell, research process of this study could be divided into three steps:

The main task in the first phase is to formulate a series of research questions and the criteria to explicitly define what is to be understood as 'visual contents'. Particularly, the graphical types contained by the school books were classified, and the classification provided a basis for the further scientific observation of their appearances in different topic areas or themes.

During phase two, the authors compiled a code scheme and determined the related representational constructs to be examined. The instrument Graphical Analysis Protocol designed by Slough et al was taken as the original coding scheme, with changes made to highlight the uniqueness of the scientific instruction in UAE. The GAP instrument provides major key perspectives that organize the representational features of graphics to be examined. There are eight categories in the scheme, that are graphic types, function, topic area, quality, ethnic representation, gender representation, type and relation to text. **Table 2** lists the research categories with detailed descriptions. The coding scheme was validated by two science education specialists. After the coding scheme being revised, the criteria for coding were discussed among the authors and a group of science teachers. Categories in the revised scheme was also checked to make sure to be relevant to the research question. Methodologically, categories were also checked to meet the requirements of being 'Exhaustive' and 'Exclusive' in a quantitative content research (Cheng and Gilbert, 2009; Eilam and Poyas, 2010). 'Exhaustive' requires every aspect of the images must be covered by the coding category. 'Exclusive' requires no overlapping between the categories.

Table 2. The coding scheme used in the study (adapted from the original scheme by Slough et al (Duit and Treagust, 2012))

Research Category		Description
I – Topic Area		
Life Science	Plant	A plant photo or drawing of indigenous (local Arabic) or foreign origin (outside the Arabic world)
	Animal	An animal photo or drawing of indigenous (local Arabic) or foreign origin (outside the Arabic world)
	Human	A human photo or drawing, female or male
Environmental Science		An environmental photo or drawing of indigenous (local Arabic) or foreign origin (outside the Arabic world)
General		A general-science photo or drawing of indigenous (local Arabic) or foreign origin (outside the Arabic world)
Earth Science		An earth science photo or drawing of indigenous (local Arabic) or foreign origin (outside the Arabic world)
Physical Science		A chemistry or physics photo or drawing of indigenous (local Arabic) or foreign origin (outside the Arabic world)
II – Graphical Types		
Iconic		Photos and pictures that depict the concrete objects in which spatial relations are isomorphic to those in the referent object.
Schematic		Diagrams abstracted from real-world entities but do not preserve the physical relations presented in the source information.
Charts and Graphs		Depict quantitative and numerical data.
Augmented Reality		'Middle ground' between completely synthetic and completely real.
III – Ethnic Representation		
Indigenous		Local photograph (gulf states or Arabic) or drawing
Foreign		Non-local photograph or drawing
IV – Gender Representation		
Female		Female human photograph or drawing
Male		Male human photograph or drawing
V – Indexing		
None		Photograph or drawing is not mentioned in text
Indexed		Photograph or drawing is mentioned in text
VI – Captioning		
No caption		No title or description under graph or drawing
Captioned		A title or description is written under graph or drawing
VII – Quality		
Dynamic		Use series of images to show change over time in graph or drawing
Static		No change with time in graph or drawing
VIII – Function		
Decoration		Does not support text, if taken out does not cause any difference in understanding of the written text
Related to Text		Important to text written, if taken out will affect understanding of text

Having agreed on the diagram coding scheme, the authors in total reviewed a sample of diagrams (approximately 10% of the total) from a range of the textbooks resulting in more than 96% agreement with the classification in order to enhance the validity and reliability of the analysis.

The encoding and data analysis were conducted in phase three. All illustrations appeared in the UAE science textbooks and practical books grade 1 to 5 were coded and analysed with the developed coding scheme. Coding of images strictly comply with the description of each research category. The visual content was analysed by taking into account the context in which the scientific concept was demonstrated before the final categorization. In order to enhance the validity and reliability of the analysis procedure, the coding result was cross-checked by two well-experienced science researchers. At last, the main author reviewed and re-entered the data and performed the statistical analysis. The visual content in the school books were calculated and the data for each category were tabulated for each chapter in every textbook and practical book. Descriptive statistics was processed by SPSS software for each variable were conducted.

RESULTS AND DATA INTERPRETATION

An overall statistics regarding the number of pages and diagrams analysed, as well as results by research questions.

(1) What are the frequencies of the graphics among the science topic areas?

According to the results shown in **Table 4**, the General Science is the scientific domain that contained the largest quantity of graphics (778), followed by 531 graphics in Life Science topic, 210 graphics were found in Environmental Science, 187 in Earth Science, and 184 in Physical Science. Meanwhile, with the increase of the years of schooling, scientific topics manifested different tendencies of graphical usage in both textbooks and practical books:

The Life Science topic is composed of three broad sub-topics. Graphics used in the topic of Plant remained relatively stable between Textbook 1 (33.3%) and Textbook 2 (32.3%), the percentage dropped sharply to 5.1% in Textbook 5; however, it was noteworthy to see that graphs regarding plants were not used in a large quantity in practical books. The percentage dropped from 3% in Practical Book 1 to 1% in Practical Book 4. With no graphics were found in Practical Book 2, Practical Book 4 and Practical Book 5. The graphics relating to Animal showed a slight fluctuation from 27.2% in Textbook 1 to 9% in Textbook 4, and lastly rose to 12.6% in Textbook 5. The percentage in Practical books showed a slight downward tendency, from 10% in Practical Book 1 to 7% in Practical Book 3. The percentage of graphics used in depicting Human showed a significant decrease, from 23.8% in Textbook 1 to 8.5% in Textbook 5. An overall increase tendency could be found in the Practical books, from 4.8% (Practical Book 1) to 12.2% (Practical Book 4).

For Environmental Science, the percentages of graphical usage demonstrated an overall fluctuation in Textbooks, with a sharp decrease from Textbook 1 (34.8%) to Textbook 3 (6.7%), and then increased to 21.4% in Textbook 5; A steady decrease of the graphic usage could be seen in Practical books, from 3.3% in Practical Book 1 to 0.4% in Practical Book 3.

For General science, the percentage of graphical usage increased slightly from 11.3% in Textbook 1 to 18.3% in Textbook 5; The percentage of graphical usage remained a relatively stable, from 3.5% (Practical Book 1) to 3% (Practical Book 5).

In Earth Science, there is an increase in the graphic usage from Textbook 1 (10.7%) to Textbook 5 (28.3%). Textbooks of Physical Science demonstrated an dramatic increase in their use of graphs, from 58.2% in Textbook 1 to 3.2% in Textbook 5; On the contrary, there was a significant increase across the practical books, from2.7% in Practical Book 1 to 6.5% in Practical Book 5.

Table 4 derived from **Table 3** and indicated graphical distribution in topic areas and from grade 1 to 5. The results show that general science topic has the largest quantity of graphics (41.2%), followed life science topic (28.1%), and then environment science (11.1%), earth science (9.9%), and lastly physical science (9.7%). However, no matter what topic area is, textbooks contain more illustrations than practical books in each grade. As for the graphics used in different grades, grade 1 has the most number of graphics (27.2%), followed by grade 3 (19.9%), grade 2 (19.5%), grade 4 (15.9%) and grade 5 (17.5%). The graphic usage differs in each grade. The most frequently used and least used types are life science (35.3%) and Earth Science (5%) in grade 1; General Science (42.4%) and Earth Science (6%) in grade 2: General science (55.7%) and Earth Science (4%) in grade 3; General Science (43.8%) and Earth Science (5.7%); and General Science (50%) and Physical Science (1.8%).

Table 3. Frequencies of graphics in various topic areas for textbooks and practical books grade 1 to 5

Grade	TOPIC AREA										Total	
	Life Science		Environmental Science		General Science		Earth Science		Physical Science			
	Freq	%	Freq	%	Freq	%	Freq	%	Freq	%	Freq	%
Grade 1	182	35.3%	80	15.5%	115	22.3%	26	5%	112	21.7%	515	27.2%
Grade 2	133	36%	53	14.4%	157	42.5%	6	1.6%	20	5.4%	369	19.5%
Grade 3	79	21%	15	4%	210	55.7%	56	14.9%	17	4.5%	377	19.9%
Grade 4	84	28.1%	17	5.7%	131	43.8%	38	12.7%	29	10%	299	15.9%
Grade 5	53	16.1%	45	13.6%	165	50%	61	18.5%	6	1.8%	330	17.5%
Total	531	28.1%	210	11.1%	778	41.2%	187	9.9%	184	9.7%	1890	100%

Table 4. Summary of graphics in topic areas and across grade 1-5

Book Type and Grade	TOPIC AREA													
	Life Science						Environmental Science		General Science		Earth Science		Physical Science	
	Plant		Animal		Human									
	Freq	%	Freq	%	Freq	%	Freq	%	Freq	%	Freq	%	Freq	%
Textbook 1	33	33.3	73	27.2	39	23.8	73	34.8	88	11.3	20	10.7	107	58.2
Practical Book 1	3	3	26	10	8	4.8	7	3.3	27	3.5	6	3.2	5	2.7
Textbook 2	32	32.3	57	21.3	30	18.3	47	22.4	143	18.4	6	3.2	17	9.2
Practical Book 2	0		14	5.2	0		6	2.9	14	1.8	0		3	1.6
Textbook 3	23	23.2	38	14	3	1.8	14	6.7	169	21.7	51	27.3	14	7.6
Practical Book 3	2	2	2	7	11	6.7	1	0.4	41	5.3	5	2.7	3	1.6
Textbook 4	1	1	24	9	39	23.8	17	8.1	104	13.4	31	16.6	17	9.2
Practical Book 4	0		0		20	12.2	0		27	3.5	7	3.7	12	6.5
Textbook 5	5	5.1	34	12.6	14	8.5	45	21.4	142	18.3	53	28.3	6	3.2
Practical Book 5	0		0		0		0		23	3	8	4.3	0	
Total	99		268		164		210		778		187		184	

(2) How various modes of graphical representations distributed in textbooks and practical books?

The graphics in all the ten primary science Textbooks and Practical Books were examined and classified into iconic, schematic, charts & graphs, and augmented reality. Analysis were then performed to see how these above mentioned representational types were used in the ten school books, either textbooks or practical books. Results about the Textbook 1 to 5 are shown in **Table 5**; Graphical usage in Practical Book 1 to 4 was summarized in **Table 6**. **Table 7** summarized the maximum and minimum used graphic types in each year of schooling.

Results showed that: iconic diagrams were the most common form of graphics. Iconic diagrams accounted for 95.6%, 89.8%, 92%, 80.8%, 82.2%, and 88.9% in Textbook 1 to 4 respectively. While schematic diagrams came second at 4.1%, 7.8%, 6.4%, 17.1%, and 15.8% in each of the four textbooks, from Textbook 1 to 4. Charts & Graphs account for 0.2%, 0.6%, 1.6%, 2.1%, and 1.7% in Textbook 1 to 4. Augmented reality type has the least amount of graphic, with 1.8% in Textbook 2 and 0.3% in Textbook 5.

As for the graphic distribution of Practical Books, a similar pattern was found as in their paired textbooks. The most common graphic form is the iconic diagrams. Its proportions account for 83.5%, 91.7%, 75%, 50%, 29% and 68.5% in Practical Book 1 to 5. With schematic diagrams at 16.5%, 5.6%, 12.5%, 33.3%, 67.7%, and 23.9% in the five Practical Books respectively. The proportions of charts & graphs are relatively low, as in 0%, 2.8%, 12.5%, 16.7%, 3.2%, and 7.6% respectively. Besides, it is noteworthy that augmented reality graphs do not appear in the five Practical Books.

Table 5. Distribution of different diagrammatic representations for grades 1 to 5 textbooks

Diagrammatic Type	Grade										Total	
	Textbook 1		Textbook 2		Textbook 3		Textbook 4		Textbook 5			
	Freq	%	Freq	%	Freq	%	Freq	%	Freq	%	Freq	%
Iconic	372	95.6	299	89.8	286	92	189	80.8	245	82.2	1391	88.9
Schematic	16	4.1	26	7.8	20	6.4	40	17.1	47	15.8	149	9.5
Charts & Graphs	1	0.2	2	0.6	5	1.6	5	2.1	5	1.7	18	1.2
Augmented Reality	0		6	1.8	0		0		1	0.3	7	0.4
Total	389		333		311		234		298		1565	

Table 6. Distribution of different diagrammatic representations for grades 1 to 5 practical books

Diagrammatic Type	Grade										Total	
	Practical book 1		Practical book 2		Practical book 3		Practical book 4		Practical book 5			
	Freq	%	Freq	%	Freq	%	Freq	%	Freq	%	Freq	%
Iconic	66	83.5	33	91.7	48	75	33	50	9	29	189	68.5
Schematic	13	16.5	2	5.6	8	12.5	22	33.3	21	67.7	66	23.9
Charts & Graphs	0		1	2.8	8	12.5	11	16.7	1	3.2	21	7.6
Augmented Reality	0		0		0		0		0		0	
Total	79		36		64		66		31		276	

Table 7. The maximum and minimum of graphic types found in each grade

Grade	Book Type			
	Textbook		Practical Books	
	Max	Min	Max	Min
Grade 1	Iconic (95.6%)	Charts & Graphs (0.2%)	Iconic (83.5%)	Schematic (16.5%)
Grade 2	Iconic (89.8%)	Charts & Graphs (0.6%)	Iconic (91.7%)	Charts & Graphs (2.8%)
Grade 3	Iconic (92%)	Charts & Graphs (1.6%)	Iconic (75%)	Schematic/Charts & Graphs (12.5%)
Grade 4	Iconic (80.8%)	Charts & Graphs (2.1%)	Iconic (50%)	Charts & Graphs (16.7%)
Grade 5	Iconic (82.2%)	Augmented Reality (0.3%)	Schematic (67.7%)	Charts & Graphs (3.2%)

Table 8. The gender representations of graphics for textbooks and practical books

Books	Gender						Total
	Male Image		Female Image		Combination		
Textbook 1	15	24.2%	37	59.7%	10	16.1%	62
Textbook 2	12	28.6%	28	66.7%	2	4.7%	42
Textbook 3	9	32.1%	19	67.9%	0	0	28
Textbook 4	18	50%	18	50%	0	0	36
Textbook 5	4	25%	10	62.5%	2	12.5%	16
Practical book 1	0		1	100%	0		1
Practical book 2	0		0		0		0
Practical book 3	0		0		0		0
Practical book 4	0		0		0		0
Practical book 5	0		0		0		0

(3) What is the distribution of the gender representations of graphics in the sampled textbooks and practical books?

The findings to the above question is summarized in **Table 8**, which shows that female images tend to appear more frequently than male images in the school books. (Female images account for 59.7% for Textbook 1, 66.7% for Textbook 2, 67.9% for Textbook 3, 50% for Textbook 4, and 62.5%for Textbook 5). In addition, Textbooks contain more human images than Practical Books (184 to 1 in quantity respectively). However, portrait of human can barely be found in Practical Books. Compared with textbooks, there is only one graph found to be containing a human figure in Practical Book 1.

Although female images tend to be more frequently selected, gender preference as appeared in the images was not obviously emphasized in science teaching and learning. In most occasions, a human image was just employed to demonstrate as human intervention or operation is required in a scientific process, like an experiment.

(4) How was the ethnic information distributed in the textbooks and practical books?

As can be seen from **Table 9**, the majority of ethnic information resides in the graphics contained by Textbooks. While practical books have less ethnical content. The indigenous information is more preferable to be used in the explanation of a concept.

The indigenous elements can be identified as people's dressing, geographical environment, climate condition etc. The 'self-centric' way of using visual displays may help eliminate students' difficulties in understanding a concept. However, ethnic identities can rarely be found in practical books. **Figure 1** shows the images coded as indigenous and foreign.

Table 9. Distribution of the ethnic information in the textbooks and practical books

Books	Ethnic				Total
	Indigenous		Foreign		
Textbook 1	104	78.8%	28	21.2%	132
Textbook 2	65	80.2%	16	19.8%	81
Textbook 3	0		0		0
Textbook 4	10	100%	0		10
Textbook 5	10	83.3%	2	16.7%	12
Practical book 1	6	60%	4	40%	10
Practical book 2	0		0		0
Practical book 3	0		0		0
Practical book 4	0		0		0
Practical book 5	0		0		0

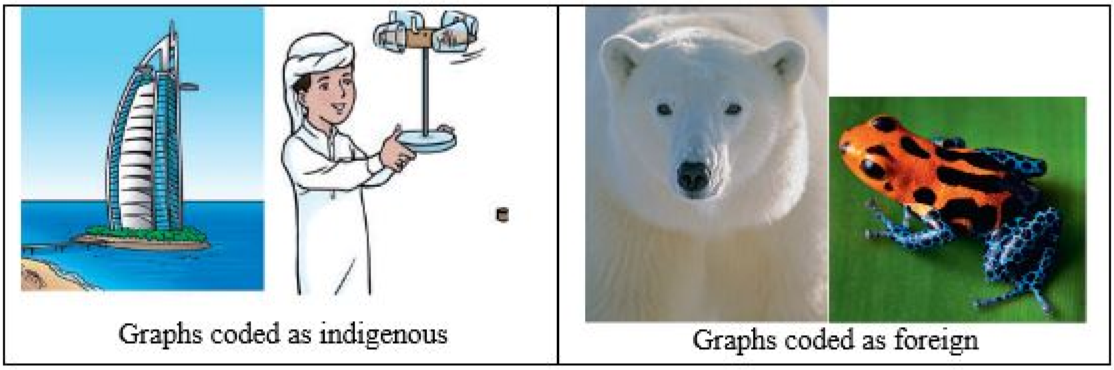

Figure 1. Graphs coded as indigenous and foreign origin

Table 10. The indexing relation between graphs and text in textbooks

Textbooks	Indexing				Total
	Not-indexed		Indexed		
Textbook 1	34	8.9%	348	91.1%	382
Textbook 2	18	5.5%	309	94.5%	327
Textbook 3	0		303	100%	303
Textbook 4	5	2.2%	227	97.8%	232
Textbook 5	24	8.1%	274	91.9%	298
Average	16.2	4.9%	292.2	95.1%	308.4

Table 11. The indexing relation between graphs and text in practical books

Practical Books	Indexing				Total
	Not-indexed		Indexed		
Practical book 1	23	29.5%	53	70.5%	78
Practical book 2	1	2.8%	35	97.2%	36
Practical book 3	3	4.5%	63	95.5%	66
Practical book 4	7	24.1%	22	75.9%	29
Practical book 5	18	58.1%	13	41.9%	31
Average	10.4	23.8%	37.2	76.2%	48

(5) How do the graphic representations relate to the text? The frequency of indexing relation between a diagram and text.

An indexing relation refers to a situation in which the diagram and the text are mutually complementary in explaining each other. As the results shown in Table 10 and Table 11, the majority of graphics were in indexing connection with the wordy information beside. In average, 95.1% of the graphics in textbooks and 76.2% of them in practical books are coded as indexed. However, the percentage of not indexed in practical books is higher than that of textbooks (23.8% to 4.9%).

Table 12. Captioning relation in the textbooks

Textbooks	Caption				Total
	Not-Captioned		Captioned		
Textbook 1	38	9.9%	344	90.1%	382
Textbook 2	50	18.4%	222	81.6%	272
Textbook 3	49	16.3%	252	83.7%	301
Textbook 4	79	34.2%	152	65.8%	231
Textbook 5	92	30.9%	206	69.1%	298
Average	61.6	21.9%	235.2	78.1%	296.8

Table 13. Captioning in the practical books

Practical Books	Caption				Total
	Not-Captioned		Captioned		
Practical Book 1	16	19.3%	67	90.7%	83
Practical Book 2	7	20%	28	80%	35
Practical Book 3	39	59.1%	27	40.9%	66
Practical Book 4	18	27.7%	47	72.3%	65
Practical Book 5	5	16.1%	26	83.9%	31
Average	85	30.4%	195	69.6%	280

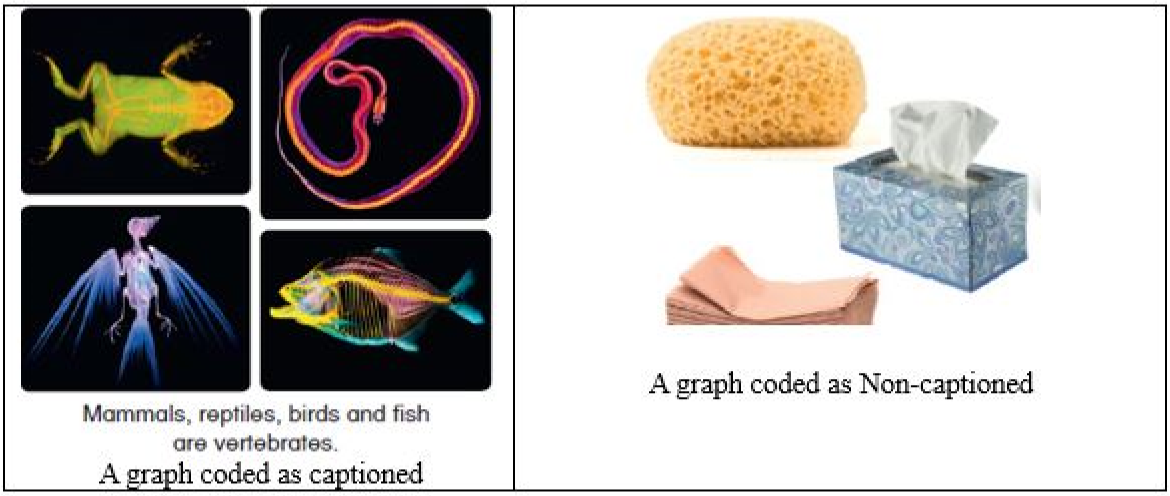

Figure 2. Captioned and non-captioned graphs

(6) How do the graphic representations relate to the text? The frequency of captioning relation of illustrations

A captioning relation refers to a brief description in written language provided to accompany the corresponding illustration. As can be seen from **Table 12** and **Table 13**, captioned illustration (78.1%, 69.6%) outnumber the not-captioned ones (21.9%, 30.4%) in textbooks and practical book categories in average.

Figure 2 shows the captioned and non-captioned graphs respectively. The caption 'Mammals, reptiles, birds and fish are vertebrates' was provided under the graphics facilitating students' understanding. However, the non-captioned graph on the right does not have any description or title underneath. These photos show various types of absorbent materials listed as examples. Without any explanations, students were asked to choose which is suitable for absorbing water.

(7) What was the quality feature (dynamic or static) of the graphical representations in the textbooks and practical books?

Table 14 and **Table 15** show the frequencies of the quality feature of graphical representations occurring in the school books. There are more static graphs than dynamic graphs in both Textbooks and Practical Books. With 79.1% are static graphs and 20.9% are dynamic graphs in Textbooks; 73.5% are static graphs and 26.5% are dynamic graphs in Practical Books.

Table 14. Frequencies of quality feature (dynamic or static) in the textbooks

Textbooks	Quality				Total
	Dynamic		Static		
Textbook 1	67	17.9%	307	82.1%	374
Textbook 2	38	11.5%	293	88.5%	331
Textbook 3	73	24.2%	229	75.8%	302
Textbook 4	61	26.3%	171	73.6%	232
Textbook 5	82	27.5%	216	72.5%	298
Average	64.2	20.9%	243.2	79.1%	307.4

Table 15. Frequencies of quality feature (*dynamic or static*) in the practical books

Practical Books	Quality				Total
	Dynamic		Static		
Practical Book 1	11	13.4%	71	86.6%	82
Practical Book 2	2	5.6%	34	94.4%	36
Practical Book 3	8	12.3%	57	87.7%	65
Practical Book 4	37	56.9%	28	43.1%	65
Practical Book 5	16	51.6%	15	48.3%	31
Average	14.8	26.5%	41	73.5%	55.8

Table 16. Frequencies of the functional connections in textbooks

Textbooks	Function				Total
	Decoration		Relation		
Textbook 1	10	2.6%	373	97.4%	383
Textbook 2	19	5.7%	313	94.3%	332
Textbook 3	0		299	100%	299
Textbook 4	1	0.4%	229	99.6%	230
Textbook 5	1	0.3%	297	99.7%	298
Average	6.2	2%	302.2	98%	308.4

Table 17. Frequencies of the functional connections in practical books

Practical Books	Function				Total
	Decoration		Relation		
Practical Book 1	3	3.6%	80	96.4%	83
Practical Book 2	0		36	100%	36
Practical Book 3	0		65	100%	65
Practical Book 4	1	1.5%	64	98.5%	65
Practical Book 5	2	6.7%	28	93.3%	30
Average	1.2	2.2%	54.6	97.8%	55.8

The majority of the graphs in both textbooks and practical books were used to convey static information. As the conceptual knowledge illustrated by 79.1% and 73.5% of the graphs were found to be static.

(8) What are the functional connections between the graphics and the written text (*decoration or relation*)?

Table 16 and **Table 17** illustrate the functional connections in textbooks and practical books. The majority of the functional connections between graphs and its related text in textbooks and practical books belong to relation function (98% and 97.8% respectively). Only a very few of the graphs were coded as the decoration function (2% and 2.2% respectively).

DISCUSSION

In this study, a descriptive statistical analysis was performed in regard with the type, function and another quality features of the graphical representations contained by UAE primary science textbooks and practical books. The results suggest that all the four types of the graphical representations were identified in the ten science books. With majority of graphical types were found to be used in all topic area. Some general features of the graphical usage could be summarized as follow:

In UAE primary science books, the general science topic (41.2%) contains the largest quantity of graphics, followed by life science (28.1%), environmental science (11.1%), earth science (9.9%), and physical science (9.7%) comes to the last (Refer to the results in **Table 3**). Prior study that have also noted the importance of using visual graphics in illustrating general science content, especially for the fundamental level of science learners. In the study, researchers examined the graphical representations in twelve Bahrain cycle-2 primary science books and workbooks, the results suggested about 70% of graphics belong to general science, 36% to life science topic area, 10% to earth science and 31% to environmental science, 3% to earth science and 31% to physical science (Ausubel, 1968). The proportions of the topic areas may vary in the school books from country to country, one general implication could be drawn is that general science is a domain in which textbook authors tend to use more visual representations to illustrate the conceptual knowledge being taught.

As for the selection of representational modes, the most frequently used visual type is the iconic, then the schematic, followed by charts & graphs, and lastly the augmented reality. This pattern applies to both textbooks and practical books. However, augmented reality graphs could rarely be found in practical books. Moreover, female images are more frequently used in describing the human intervention of a scientific process, for example, a scene in which female teachers are doing experiments with (female) students. In addition, textbook authors tend to have more indigenous elements in the explanation of concepts. Compared with the foreign elements, the indigenous elements are more preferable to be used in the graphical design. The above finding also correlates with a recent study (Vygotsky, 1978), in which iconic diagrams were identified as the most frequently used diagram type, the least used diagram type is Chart & Graphs, Augmented Reality graphics were rarely used in primary workbooks in another middle east country. It is also noteworthy seeing researchers of earlier studies also emphasized the importance of teaching diagrammatic literacy to improve students' learning of science concepts (Gilbert, 2007; Liu and Treagust, 2013; Novick, 2006). An iconic diagram was defined as an accurate depiction of concrete objections in which spatial relations in the diagram are isomorphic to those in the referent object (Piaget, 1971). The results of this study found empirical evidences supporting the advantages that iconic graphs have in the visual cognitive process. In particular, iconic diagrams are more often used in the beginning science learning in which is omorphic interplay need to be built between the graphic depiction and the concrete referent objects.

As for the relations between graphical and textual representations, the coding scheme summarized information into three aspects: *Indexing* – all most all of the graphs and text were mutually explaining each other (95.1% for textbooks and 76.2% for practical books); *Captioning* – most of the graphics were captioned, which means, a brief textual description was provided; *Functioning* – the logical and semantic relation between the visual and textual representations were further classified as being decorational or relational. The data shows the functioning relation belongs to the relational type, that is, the graphs were not simply provided to be appealing to the teachers and students. Though conceptual frameworks were proposed for depicting the multiple representational learning (Bell, 2001, p. 14), these frameworks depicts the leaners' cognitive process in such a subtle manner that can hardly to be applied in book analysis studies. Especially when voluminous of graphical and textual representations are posed together, which could deteriorate the reliability of coding. One of another obvious findings to emerge from this study is engaging more than one mode of representations in its coding scheme, the three classified functions (*Indexing, Captioning,* and *Functioning*) would be more suitable for book studies.

This study has certain limitations that could be considered in future research:

(a) In addition to the results from quantitative analysis, textbook users' (teacher/students) opinions could have been sought. Their opinions might provide a brand new perspective viewing the graphical usage. The actual uses of different modes of visual representations were observed especially in real science instructional context, the observations were considered helpful in triangulating the findings.

(b) The analysis of graphics could also have included items regarding specific drawing conventions, such as colouring, arrows and etc. Though those visual conventions alone are not sufficient for students to make sense of the information in graphics, knowing how learners refer to those conventions may also provide a means of analysing textbook usage of graphics.

(c) There may be more properties inherent in the graphical representations that should have been included in the coding scheme. Especially when new types of visual representations emerge along with the development of the printing technology, countries all over the world have different science teaching context. Those features may of great value to the improvement of the coding instrument.

Future studies could investigate (a) the cognitive relationships between different modes of graphic representations and other representational modes such as text, (b) the instructional use of different graphics in classroom settings, and (c) students' learning efficacies when reading different modes of graphics.

CONCLUSIONS

Previous studies into content analysis of science textbooks about visual representation usage focused on characterizing different functional features of visualizations by examining the relationship among the visual displays as well as teachers' instructional practices (2010). However, this study did not aim to investigate students' exact conceptual learning via different modes of visual displays. Guided by the improved coding scheme, this study set out to determine the patterns in which graphical representations distributed in the UAE primary science books.

The analysis of the ten science books may provide science educators with insights on the distributional manner that graphic representations have been organized in these books.

Research findings of this study could generally be divided into three aspects regarding the graphic usage:

One of the most significant findings to emerge from this research is analysing the graphical distributions among a number of scientific topic areas. The results shed light on the general visual usage of the science textbooks and practical books. However, the frequency does not equal to the level of importance of each subject area. In addition, it is neither ideal nor possible for all topic areas to have equivalent number of graphs. To some degree, the finding echoes UAE primary science curriculum, in which content areas like general science and life science might play a pivotal role in the science learning.

The second research objective moves on to the selection of graphical types in the illustration of domain knowledge. Various modes of graphical representations were found in the textbooks and practical books of each grade. The overall graphical selection was in accordance with the primary level of science teaching and learning, as iconic graphs are advantageous in presenting the information that is more tangible, concrete, and intelligible to the learners' background knowledge. It is also noteworthy to see that the higher the grade of science learning, the more frequently schematic graphs to be used. Human images were more likely to be used for instructional purpose, while less portraits of human were found in practical books. It is also observed that female images tend to appear more frequently than male images in the school books.

The visual illustrations in the ten UAE science books were also found to be accommodating to both indigenous and foreign elements. The indigenous graphs outnumber the foreign ones, as primary level of science teaching might require learners' interpretation of the concepts to be taught and their prior knowing.

Some other strengths that UAE textbooks have in facilitating students' learning. Such as, many items in practical books that need students to draw. Students' drawing may not only sever as a method to demonstrate their learning, it could also be a way for evaluating the individual conceptual learning. In textbooks, the graphical and textual representations supplement each other through description of the scientific facts as well as raising questions. In other words, graphs are always companied by a short passage entitled "Fact" describing the content of the image in an objective way. Besides, some questions may also be provided for the guidance of students' interpretation. The "Fact + Questions" pattern would help the book authors skilfully avoid the religious controversies that students may encounter during the science learning process.

Although an extent of the graphs were designed without caption, the function and content of the graphs were in consistent pattern. The topic may exert a guidance towards student's understanding of the graphs, because students may perceive its possible implication.

The coding scheme may not be able to serve as a universal instrument for classifying and analysing the graphical representations contained in textbooks. For more items could be added to fit the actual and indigenous science learning materials. However, findings summarized in the study provide suggestions that teachers should evaluate the visual material before its application in the classroom teaching. It also implies the importance of graphic usage that authors and curriculum developers should take into account when developing educational material. Nevertheless, in terms of methodological aspects, this study also provides empirical evidences to develop the 'Coding Scheme' that could be used for further textbook studies regarding visual usage. It is hoped that more items could be included into the original coding instrument Graphical Analysis Protocol (GAP) by Slough et al. An improved instrument could be used to identify and analyse more and other features of graphical representations or especially the interactions between different modes of representational learning and representational attributes. For instance, colour vs. black printing, the drawing and the graphical compositions – lines, arrows, shapes and etc.

Previous studies also tend to analyse two or more modes of representations or a number of spatial conventions under one representation that could promote conceptual learning (Khine, 2013). This study also provided practical advice for teachers and textbook authors when including different modes of visual representations in the everyday science teaching. Without any doubt, well-designed and visual-friendly learning materials are believed to exert a positive effective in helping students understand the difficult concepts and to avoid misconceptions. This study is considered of a benefit to teachers, textbook authors and curriculum developers in producing reform-oriented science textbooks, which would help in improving the quality of science learning in UAE. Although a large number of textbook studies in the literature focused on the graphics used in secondary and tertiary level of science education, preliminary textbooks deserve more attention.

REFERENCES

Abd-El-Khalick, F., Myers, J. Y., Summers, R., Brunner, J., Waight, N., Wahbeh, N., . . . Belarmino, J. (2017). A longitudinal Analysis of the extent and manner of representations of nature of science in U.S. high school biology and physics textbooks. *Journal of Research in Science Teaching*, 54(1), 82-120. https://doi.org/10.1002/tea.21339

Ausubel, D. P. (1968). *Educational psychology: A cognitive view*. New York: Holt, Rinehart & Winston, Inc.

Azuma, R. T. (1997). A survey of augmented reality. *Presence*, 6(4), 355-385. https://doi.org/10.1162/pres.1997.6.4.355

Bell, P. (2001). Content analysis of visual images. In T. V. Leeuwen and C. Jewitt (Eds.), *Handbook of visual analysis* (pp. 14). Thousand Oaks, CA: Sage.

Chamblisss, M. J. and Calfree, R. C. (1989). Designing science textbooks to enhance students' understanding. *Educational Psychologiest*, 24(3), 307-322. https://doi.org/10.1207/s15326985ep2403_5

Cheng, M. and Gilbert, J. K. (2009). Gilbert, J. K. Towards a better utilization of diagrams in research into the use of representative levels in chemical education. In J. K. Gilbert and D. F. Treagust (Eds.), *Multiple representations in chemical education* (Vol. 4). The Netherlands: Springer. https://doi.org/10.1007/978-1-4020-8872-8_4

Devetak, I. and Vogrinc, J. (2013). The criteria for evaluating the quality of the science textbooks. In M.S. Khine (Ed.). *Critical analysis of science textbooks: Evaluating instructional effectiveness* (pp. 3-15). Dordrecht, The Netherlands: Springer. https://doi.org/10.1007/978-94-007-4168-3_1

Duit, R. and Treagust, D. F. (2012). How can conceptual change contribute to theory and practice in science education? In B. J. Fraser (Ed.), *Second international handbook of science education* (pp. 107-118). Netherlands: Springer. https://doi.org/10.1007/978-1-4020-9041-7_9

Eilam, B. and Poyas, Y. (2010). External visual representations in science learning: The case of relations among system components. *International Journal of Science Education*, 32(17), 2335-2366. https://doi.org/10.1080/09500690903503096

Gilbert, J. K. (2007). Visualization: A metacognitive skill in science and science education. In J. K. Gilbert (Ed.), *Visualization in science education* (pp. 9-28). Dordrecht, The Netherlands: Springer.

Hegarty, M., Carpenter, P. A. and Just, M. A. (1991). Diagrams in the comprehension of scientific texts. In R. Barr, M. L. Kamil, P. Mosenthal and P. D. Pearson (Eds.), *Handbook of reading research* (Vol. 2, pp. 641-668). New York: Longman.

Irez, S. (2010). Nature of science as depicted in Turkish biology textbooks. *Science Education*, 93(3), 422-447. https://doi.org/10.1002/sce.20305

Izquierdo, M. and Gouvea, G. (2008). A proposal for textbook analysis: Rhetorical structure. *Science Education International*, 19(2), 209-218.

Kahveci, A. (2010). Quantitative Analysis of Science and Chemistry Textbooks for Indicators of Reform: A complementary perspective. *International Journal of Science Education*, 32(11), 1495-1519. https://doi.org/10.1080/09500690903127649

Khine, M.S. (2013). Analysis of science textbooks for instructional effectiveness. In M.S. Khine (Ed.), *Critical Analysis of Science Textbooks: Evaluating instructional effectiveness* (pp. 303-310). Dordrecht, The Netherlands: Springer. https://doi.org/10.1007/978-94-007-4168-3_15

Kloser, M. (2016). Alternate text types and student outcomes: an experiment comparing traditional textbooks and more epistemologically considerate texts. *International Journal of Science Education*, 38(16), 2477-2499. https://doi.org/10.1080/09500693.2016.1249532

Koppal, M. and Caldwell, A. (2004). Meeting the challenge of science literacy: Project 2061 efforts to imporve science education. *Cell Biology Education*, 3, 28-30. https://doi.org/10.1187/cbe.03-10-0016

Lee, V. R. (2010). Adaptation and continuities in the use and design of visual representations in US middle school and design of visual representations in the US middle school science textbooks. *International Journal of Science Education*, 32(8), 1099-1126. https://doi.org/10.1080/09500690903253916

Liu, Y. and Khine, M. S. (2016). Content Analysis of The Diagrammtic Representations of Primary Science Textbooks. *Eurasia Journal of Mathematics, Science & Technology Education*, 12(8), 1937-1951. https://doi.org/10.12973/eurasia.2016.1288a

Liu, Y. and Treagust, D. F. (2013). Content Analysis of Diagrams in Secondary School Science Textbooks. In M. S. Khine (Ed.), *Critical Analysis of Science Textbooks* (pp. 287-300). Dordrecht, The Netherlands: Springer. https://doi.org/10.1007/978-94-007-4168-3_14

Martinez-Gracia, M. V., Gil-Quilez, M. J. and Osada, J. (2006). Analysis of molecular genetics content in Spanish secondary school textbooks. *Journal of Biological education*, 40(2), 53-60. https://doi.org/10.1080/00219266.2006.9656014

Muspratt, S. and Freebody, P. (2013). Understanding the disciplines of science: Analysing the language of science textbooks. In M. S. Khine (Ed.), *Critical Analysis of Science Textbooks* (pp. 33-59). Dordrecht: Springer. https://doi.org/10.1007/978-94-007-4168-3_3

Ninnes, P. (2000). Representations of indigenous knowledge in secondary school science textbooks in Australia and Canada. *International Journal of Science Education*, 22(6), 603-617. https://doi.org/10.1080/095006900289697

Novick, L. R. (2006). The importance of both diagrammatic conventions and domain-specific knowledge for diagrams literacy in science: The hierarchy as an illustrative case. In D. Barker-Plummer (Ed.), *Diagrammatic representation and inference* (Vol. 4045/2006, pp. 1-11). Berlin/Heidelberg: Springer. https://doi.org/10.1007/11783183_1

Orgill, M. (2013). How effective is the use of analogies in science textbooks. In M. S. Khine (Ed.), *Critical Analysis of Science Textbooks* (pp. 79-99). Dordrecht: Springer. https://doi.org/10.1007/978-94-007-4168-3_5

Piaget, J. (1971). *Science of Education and the Psychology of the Child*. New York: Viking.

Pingel, F. (2010). *UNESCO Guidebook on Textbook Research and Textbook Revision*. Paris: United Nations Educational, Scientific and Cultural Organization.

Polikoff, M. S., Zhou, N. and Campbell, S. E. (2015). Methodological choices in the content analysis of textbooks for measuring alignment with standards. *Educational Measurement: Issues and Practice*, 34(3), 10-17. https://doi.org/10.1111/emip.12065

Pozzer, L. L. and Roth, W. M. (2003). Prevalence, function, and structure of photographs in high school biology textbooks. *Jounral of Research in Science Teaching*, 40(10), 1089-1114. https://doi.org/10.1002/tea.10122

Renkl, A. and Scheiter, K. (2017). Studying visual displays: How to instructionally support learning. *Educational Psychology Review*, 29(3), 599-621. https://doi.org/10.1007/s10648-015-9340-4

Rillero, P. (2013). A content analysis of science in nineteenth century US readers: Early American science education. In M. S. Khine (Ed.), *Critical Analysis of Science Textbooks* (pp. 161-172). Dordrecht: Springer. https://doi.org/10.1007/978-94-007-4168-3_8

Slough, S., McTigue, E., Kim, S. and Jennings, S. (2010). Science textbooks' use of graphical representations: A descriptive study of four sixth grade science texts. *Reading Psychology*, 31, 301-325. https://doi.org/10.1080/02702710903256502

Strike, K. A. and Posner, G. J. (1982). Conceptual change and science teaching. *International Journal of Science Education*, 4(3), 231-240. https://doi.org/10.1080/0140528820040302

Vekiri, I. (2002). What is the value of graphical displays in learning? *Educational Psychology Review*, 14(3), 261-312. https://doi.org/10.1023/A:1016064429161

Vygotsky, L. S. (1978). *Mind in society: The development of higher psychological processes*. Cambridge, MA: Harvard University Press.

Wiseman, A. W. (2014). Representations of Islam and Arab societies in western secondary textbooks. *Digest of Middle East Studies*, 23(2), 312-244. https://doi.org/10.1111/dome.12047

Yasin, M. S. M., Hamid, B. A., Othman, Z., Bakar, K. A., Hashim, F. and Mohti, A. (2012). A visual analysis of a Malaysian English school textbooks: Gender matters. *Procedia - Social and Behavioral Science*, 69, 1871-1880. https://doi.org/10.1016/j.sbspro.2012.12.140

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Children Have the Capacity to Think Multiplicatively, as long as …

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ABSTRACT

Multiplicative thinking has been widely accepted as a critically important 'big idea' of mathematics and one which underpins much mathematical understanding beyond the primary years of schooling. It is therefore of importance to consider the capacity of children to think multiplicatively but also to consider the capacity of their teachers to teach multiplicative thinking in a conceptual manner. This article focusses specifically on the conceptual links between the multiplicative array, the notion of numbers of equal groups in the multiplicative situation, factors and multiples, the commutative property of multiplication, and the inverse relationship between multiplication and division. A study involving a large sample of primary school students found that whilst most students demonstrated an understanding of some of the aforementioned elements, hardly any of the students were able to connect the ideas or to explain them in terms of each other. As a consequence of the findings, the impact of teacher knowledge on children's capacity to think multiplicatively was considered.

Keywords: multiplicative thinking, connected knowledge, teacher knowledge, arrays, factors, commutativity, inverse relationship, multiplication, division

INTRODUCTION

Multiplicative thinking is acknowledged as a 'big idea' of mathematics (Hurst and Hurrell, 2014; Siemon, Bleckley and Neal, 2012) which underpins much of the mathematics learned beyond the middle years of primary/elementary schooling. Aspects of multiplicative thinking, multiplication, and division have been variously described by mathematics researchers and educators. However, few people have offered a definition that adequately covers the broad scope of multiplicative thinking. The following definition, based on the work of Siemon, Breed, Dole, Izard and Virgona (2006) and Siemon, Bleckley and Neal (2012) is offered here. Multiplicative thinking is demonstrated by an ability to

- Work flexibly with a wide range of numbers including very large and small whole numbers, decimals, fractions, ratio and percentage;
- Work conceptually with the relative magnitude of whole and decimal numbers in a range of representations, demonstrating an understanding of the notions of 'times bigger' and 'times as many';
- Demonstrate a conceptual understanding of the multiplicative situation, the relationship between multiplication and division, numbers of equal groups, factors and multiples, and the various properties of multiplication; and
- Articulate a conceptual understanding of a range of multiplicative ideas in a connected way with explicit language and terminology.

It is evident from the definition that multiplicative concepts are complex but connected and this will be explored shortly. However, while this article explores the extent to which primary/elementary students have the capacity to think multiplicatively, there is a key factor at play – the mathematical content knowledge of teachers – and whether that content knowledge is held in a connected and conceptual way. Thanheiser, Philipp, Fasteen, Strand and Mills (2013, p. 137) note that '. . . teachers' mathematical content knowledge continues to be a major area of concern' despite the best efforts of universities and professional learning providers to offer courses specifically designed to develop teacher knowledge. Thanheiser et al. (2013, p. 137) also noted that

> "the fact that most PSTs [Pre-Service Teachers] and teachers do not understand the rationales behind the procedures they teach is a major concern for those of us responsible for teaching PSTs".

Whilst Thanheiser et al. reported on a study of PST knowledge, it seems reasonable to suggest that the same situation might apply to teachers already serving in classrooms. This raises an important question. If teachers hold their content knowledge in an unconnected and procedural way, might that inhibit students' ability to make multiplicative connections and thus develop a conceptual rather than procedural view of multiplicative ideas? This article essentially focuses on the capacity of students to think multiplicatively but it necessarily follows that teacher knowledge will be part of that discussion.

BACKGROUND

As previously mentioned, this article explores several ideas within the multiplicative situation – the multiplicative array, factors and multiples, commutativity, numbers of equal groups, and the inverse relationship between multiplication and division. Hence, it is essentially about understanding the structure of the mathematics, and to find out if primary school students can see the connections between those ideas. This then demands consideration of teacher knowledge. In the following overview, although the five main listed ideas are considered individually, inevitably, there is a degree of crossover as the ideas are inexorably connected with one another.

Arrays and Equal Groups

In his article reporting on a group of middle school teachers engaging in 'concept study', Davis (2008) points to the importance of conceptual understanding and implies that teachers need to have a deep knowledge of key concepts such as multiplication. As part of the 'concept study' and prior to their group meetings, teachers involved their students in discussions around ideas such as 'What are factors and products?' and 'What is division?' They also considered ideas like 'What are some of the different ways you can talk about multiplication? How might you draw it?' (Davis, 2008, p. 91). The last two questions allude to a strong case for the use of the array (the merits of which will be discussed in more detail) as a model for understanding the multiplicative situation. The sort of classroom activity described by Davis goes to the heart of what could be considered to be the key issue here – teachers needing to hold a deep and connected understanding of multiplicative concepts and be able to make the connections explicit to their students (Charles, 2005; Clarke, Clarke and Sullivan, 2012; Hurst, 2015). Knowing about factors and products, and the array as a representation of the multiplicative situation, are significant parts of that.

The central importance of the array has been described by others including Jacob and Mulligan (2014), Young-Loveridge (2005), and Young-Loveridge and Mills (2009). Askew's (2016) discussion of the array underlines an important aspect of the connections that need to be realised, identified and understood by children. He says that teachers need to assist children to move 'from using them as models *of* to being models *for* and then to becoming *tools for thinking with*' (Askew, 2016, p. 139). In other words, teachers need to make the connections between the array and ideas such as factors, equal groups, the inverse relationship between multiplication and division, and the development of the algorithm from the grid method. Askew (2016) supports this, noting that the array supports the understanding of connections between multiplication and division.

It is Askew's (2016) point that the array needs to be much more than a symbolic representation and more of 'a tool for thinking with' that is of interest. It is essential that the array is not something that is briefly introduced and 'covered' superficially. Rather the array needs to part of the language of the mathematics classroom at all times and used as a key tool for helping children understand the structure of the multiplicative situation. For this to occur, teachers need to understand how and why the array is important.

Matney and Daugherty (2013) are very clear about the power of the array to help students visualize mathematical structures like commutativity and distributivity, adding that students' computational errors may be compounded by their lack of understanding of what they are doing. They note that students are better able to understand commutativity when they rotate the array through ninety degrees. However, more important is that they encourage students to "consider why it might be advantageous to think of 4 x 7 and 7 x 4 as different, even though the product is the same through commutativity" (Matney and Daugherty, 2013, p. 151-152). In terms of mathematical structure, it is important to understand that the product in both cases is the same. However, being able to understand and articulate that four groups of seven is quite a different multiplicative situation to seven groups of four, demonstrates more powerful knowledge.

Factors, Factorisation, and Divisibility

Feldman (2014) discusses activities involving prime factorisation of numbers as an important part of number theory. He notes that an understanding of factors helps students to ". . . identify the relationship between a factor and its multiples and make sense of important properties such as the distributive, associative, and commutative properties" (Feldman, 2014, pp. 231-232). Rather than students simply being able to identify factors, one consequence of a deep understanding of factors is that students are able to work more flexibly and efficiently with a range of numbers (Feldman, 2014) which supports the definition of multiplicative thinking provided earlier. Feldman also notes that, despite the general paucity of research into middle school students' mathematical understanding, there is sufficient evidence to show that a strong understanding of factors leads to a better understanding of key algebraic concepts.

This is supported by some earlier work from Zazkis and Campbell (1996). In discussing "the encapsulation of divisibility", they note that many of the participants in their study (in this case pre-service teachers) had not connected divisibility with multiplication, factors, or distributivity, saying that there is a need for an explicit pedagogical approach to the inverse relationship (p. 562). Further, they make a powerful point that

> If one is to meaningfully continue in mathematics, the basic concepts of arithmetic must be grasped. If this is not happening in the middle grades, then it should come as no surprise that many students fail to make a successful transition to algebra. We believe that developing a conceptual understanding of divisibility and factorization is essential in the development of conceptual understanding of the multiplicative structure of numbers (Zazkis and Campbell, 1996, p. 562).

They imply that, rather than teach multiplication and division and multiplication separately, there is a need to consider the 'multiplicative situation' as a whole and make explicit connections between ideas like factorisation, properties of multiplication, and the inverse relationship.

Young-Loveridge and Mills (2009) studied the use of 'dotty arrays' with a group of Year 7 and 8 students (aged 12 and 13 years), and found that the array was helpful in supporting students' understanding of the algorithm for multiplying two 2-digit numbers. They specifically use the term 'factor' in describing how the array was useful noting that students needed to be able to partition the two digit factors and also that dotty arrays helped students to visualise the rectangular grid representation of multiplication (Young-Loveridge and Mills, 2009). Young-Loveridge and Mills also cite the New Zealand Numeracy Development Project (Ministry of Education, 2007) materials that multiplicative thinking involves, amongst other things, the construction and manipulation of factors. This seems to imply that the language of factors ought to be explicitly used with students to consolidate the connection with arrays.

Commutativity

Anthony and Walshaw (2000) found that students aged 9 and 13 years struggled to articulate an understanding of commutativity and did not display any real depth of multiplicative thinking. In particular, they noted that students were unable to recognize the connection between the array and the commutative property of multiplication and despite being able to engage in discussion, were very imprecise in terms of explaining commutativity. Students used terms like 'switch arounds' and 'turnarounds' and a range of similar terms but could not actually explain why the commutative property worked in terms of a model such as the array, or the language of factors and multiples. They stated it in these terms:

> "None of the sampled students appeared to construct and coordinate composite units through an array – visualizing the array in two unique orientations – that would be suggestive of reasoning multiplicatively and thinking in commutative terms" (Anthony and Walshaw, 2000, p. 97).

Warren and English (2000) arrived at a similar conclusion following a study of a group of 10-12 year old students. Whilst almost all of the students (95.7%) could identify the commutative property, only 60.6% of them could generate another example of it, and only 13.8% could generate two further examples. As well, they found that only 17% of students could offer a valid explanation but this was generally in terms of 'switching the numbers around' and not in terms of the array or factors and multiples. In noting that this is indicative of students not understanding the structure of the mathematics, they stated that this lack of understanding is largely due to the use of computational procedures rather than the exploration of mathematical relationships (Warren and English, 2000).

Squire, Davies and Bryant (2004) also noted the importance of students developing a conceptual understanding of the mathematics that underpins multiplication facts and algorithms. They note a number of benefits from students developing a strong conceptual understanding including greater flexibility in their thinking, working quicker, deriving unknown facts from known facts, and being more efficient at solving problems (Squire et al., 2004). They discuss the results of a study showing students being more efficient at identifying and using the commutative property than the distributive property but at no stage were children asked to consider why the property works. Indeed, in noting the underlying importance of commutativity, they added that it is vital for educators to consider how to develop children's understanding of commutativity (Squire et al., 2004). This must go beyond describing in terms of a 'switch around' as discussed by Anthony and Walshaw (2000). Baroody (1999, p. 184) also indicated the importance of the commutative property in terms of students being able learn multiplication facts noting that if students 'did not understand the commutative relationship . . . they would have lacked the means to generalize their learning'. The key word here is 'understand', again at a more significant level than simply identifying the commutative property or describing in terms of 'turn arounds'. Holding knowledge that the order can be changed is at a lower level than understanding why the same product is obtained.

Larsen's (2010) work with a small group of undergraduate students appears to support this. He states that a likely reason for students experiencing difficulties with the associative and commutative properties may be that too much imprecise and informal language is used in classrooms to describe these properties (Larsen, 2010). Again, there is a strong indication that teachers need to understand the structure of the mathematics around the commutative property and explicitly use terminology such as 'factors and multiples' and the array as a model for understanding the property, rather than allow students to continue to discuss commutativity in terms of 'switch arounds' and the like.

Numbers of Equal Groups and the Inverse Relationship

In their discussion of the importance of the array, Jacob and Mulligan (2014) focus on the structure of the multiplicative situation based on numbers of equal groups, beginning in the early years of schooling. They write that the structural relationship between multiplication and division needs to be emphasised before formal recording with symbols is introduced. As well, students need to be able to think simultaneously about all three quantities – the total, the number of groups and the number in each group – and more importantly, use the associated mathematical language of factors and multiples. However, they note that, unless teaching is explicit, '. . . the relationship between multiplication and division may go unnoticed by many students. The array can be used to focus students' attention on that relationship' (Jacob and Mulligan, 2014, p. 37). Jacob and Mulligan also point out that an understanding of commutativity can be greatly enhanced by the use of the array.

Siemon (2013) goes further and points out that the traditional approach is to teach multiplication before division whereas children are accustomed to the notion of sharing or splitting into equal groups, which is where the early emphasis should lie. Her example of the use of the nursery rhyme, 'Baa Baa Black Sheep', when a teacher asked a class of five year olds what would happen if there were five sheep, is quite powerful. Siemon (2013, p. 45) indicates that

> "This is essentially a ratio or times as many idea [e.g., 3 times as many bags of wool as sheep] and is quite distinct from the equal group idea".

The 'times as many' notion needs to be explicitly linked to the language of the multiplicative situation to develop a rich understanding of the inverse relationship. Siemon (2013) also underlines the importance of the array model as a basis for understanding multiplication with larger numbers and implies that the language of factors and multiples needs to be explicitly used to ensure that students' understanding of the multiplicative situation is allowed to fully develop. The language of factors and multiples, 'times bigger', and 'scaling up and down' are important tools for mathematizing – reasoning in a mathematical manner through having the language tools to consider or articulate ideas mathematically.

Downton (2013) used a sample of eight year old students and explored their methods of solving a range of division problems, rather than focus on whether the problem was based on partitive or quotitive division. She found that students intuitively used multiplication strategies to solve the division problems, irrespective of whether the problems were partitive or quotitive in nature. Downton suggests that it is the relationship between multiplication and division and the use of a rich variety of problem types in the early years that is a key factor in helping students to think multiplicatively. In an earlier article, Downton (2008) also said that students needed to experience a range of semantic structure and contexts in solving multiplicative problems, noting again the importance of the relationship between multiplication and division and the associated language before any use of symbols or formal recording is introduced.

Connected Knowledge

In an editorial column, Gojak (2013) considered the role of elementary teachers, noting that the aim of classroom instruction needs to be to develop deep conceptual understanding. She concluded that if students are to develop such solid conceptual understanding of mathematics, then it follows that their teachers must deeply understand those mathematical concepts (Gojak, 2013). The implication is that this 'deep understanding' includes a knowledge of how the 'big ideas' such as multiplicative thinking are made up of many other ideas, the connections within and between which must be deeply understood by teachers. This is supported by the work of others such as Chick, Pham, and Baker (2006) in developing a framework for analysing pedagogical content knowledge, albeit in the context of a study about the subtraction algorithm. In particular, they referred to teachers' ability to identify the critical components of a concept so that they are able to help their students understand and apply that concept (Chick et al., 2006). It could be said that the connections between the five ideas described earlier (the multiplicative array, factors and multiples, commutativity, numbers of equal groups, and the inverse relationship between multiplication and division) is an example of such connected knowledge needed by teachers.

Since Skemp's (1976) seminal article on the merits of teaching for relational understanding as opposed to instrumental understanding, others have made similar strong statements about the need for teachers to hold a broad and deep connected knowledge of mathematical concepts. Ma (2010) described this in terms of 'Profound Understanding of Fundamental Mathematics' (PUFM) and 'knowledge packages', which could be described as representations of connections between ideas. Despite there being widespread acknowledgement of the worth of such work, there is also plenty of evidence to suggest that many teachers do not hold knowledge for teaching mathematics in those ways. For example, following a study of four primary teachers and their classes, Mills (2015, p. 435) concluded that

> "[the] teachers seldom exhibited a deep and thorough conceptual understanding of aspects of the mathematics they were teaching . . . [which] contributed to their confusion within the key mathematical concepts they were teaching, and the significance of consistently using correct mathematical language".

Mills (2015) also stated that teachers did not make connections between ideas, nor did they display strong number sense.

Sullivan (2008) underlines the vital importance of teachers holding their mathematical knowledge in a connected way. He implies the necessity for teaching to be explicit and to highlight the ways in which ideas are connected as 'the connection between the representations and the concepts may be difficult for students to ascertain for themselves' (Sullivan, 2008, p. 7). This supports the findings of an earlier study by Askew, Brown, Rhodes, Wiliam and Johnson (1997) which determined that the most effective teachers of numeracy were those who held a 'connectionist' view of mathematical knowledge.

METHODOLOGY

The study reported on here is part of an on-going project into children's multiplicative thinking. The research questions under consideration are:

- To what extent do children in Years 4, 5 and 6 (i.e., of ages 9, 10, and 11 years) think in a connected way to explain multiplicative concepts?
- What is the extent of teacher knowledge of multiplicative concepts?

Multiplicative thinking was defined earlier and this article is concerned with two parts of that definition:

- Demonstrate a conceptual understanding of the multiplicative situation, the relationship between multiplication and division, numbers of equal groups, factors and multiples, and the various properties of multiplication
- Articulate a conceptual understanding of a range of multiplicative ideas in a connected way with explicit language and terminology.

Data have been generated from two sources – a written Multiplicative Thinking Quiz (MTQ), and some teacher interviews. These instruments have been developed over the three and half year life of the project. The student sample (n=545) for this article is drawn from 17 Western Australian classes and two classes in the south-west of England and consists of 167 Year 4 students (aged 9 years), 205 Year 5 students (aged 10 years), and 173 Year 6 students (aged 11 years). Responses from the MTQ were manually entered onto an Excel spreadsheet and correct responses were denoted by a '1' with incorrect responses given a '0'. Tallies were then generated using the 'sum' tool.

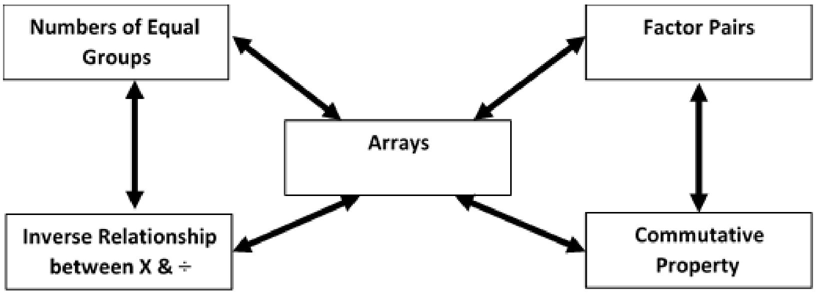

Figure 1. Conceptual Framework

Table 1. MTQ questions related to the conceptual framework

Target Idea	Wording of Question	Responses Sought
Numbers of equal groups	1(b) What do the numbers in 8 x 7 tell you? 1(c) Write a story (a word problem or a story problem) that shows what 8 x 7 means.	States that 8 means the number of groups and 7 means the number in each group. Writes a story problem that reflects numbers of equal groups.
Arrays	2(a) Draw a picture (using dots or crosses) that shows the expression 4 x 3	A 'tight' array is drawn with only small gaps between items.
Arrays	2(b) Can you draw it in another way?	Drew a rotated array
Factors	3 (iii) Write as many different multiplication sentences as you can that will give you an answer of 24	Writing all factors for 24 Writing factors as commutated factor pairs
Factors	4(a) (iii) Write as many factors of 30 as you can.	Writing all factors for 30 Writing factors as commutated factor pairs
Commutative Property	7(a) Which of the following will give you the same answer as 6 x 17? (16 x 7) (17 x 6) (7 x 16) 7(b) Give a reason for two of your choices of 'Yes' or 'No' answers	Identifies the expression which demonstrates the commutative property. Provides an explanation based on equal groups, or factors, or the rotated array.
Inverse relationship between multiplication and division	8(a) If you know that 24 x 6 = 144, which of the following must you be able to work out from that? (144 ÷ 6) (144 x 6) (144 ÷ 24) (144 x 24) 8(b) Give a reason for two of your choices of 'Yes' or 'No' answers.	Identifies expressions which reflect the inverse relationship. Provides an explanation based on factors, multiples, or products, or the array.

The conceptual framework for this study is based on part of a draft model for multiplicative thinking which is underpinned by the earlier background discussion. The framework draws on the connections between the five aspects of multiplicative thinking already discussed and is shown as **Figure 1**.

The conceptual framework is based on several propositions – that the commutative property can be understood and explained using arrays and factor pairs; the inverse relationship can be understood and explained using arrays and the concept of equal groups (as well as factors); and that the associated mathematical language needs to embedded in classroom discourse to enable students to articulate their thinking. Questions around the five ideas were developed and included in the MTQ. The questions were designed to identify whether or not students knew about aspects of multiplicative thinking, but more importantly, if they could articulate a clear and connected understanding of them based on mathematical structure. **Table 1** shows the specific questions as they relate to each of the five aspects of **Figure 1** along with the particular responses sought by the research team. The wording of each question was kept reasonably open to see what students' informal responses would be. For instance, the wording of Questions 3(iii) and 4(a) (iii) did not make mention of 'pairs' because it was important to see if students informally wrote the factors in pairs and indeed if they wrote them in 'commutated pairs' (i.e., 8 x 3 and 3 x 8). When the MTQ was administered, each question was read out and students were given a reasonable amount of time to write an answer before the next question was read. This was done to minimise any potential difficulties which any students may have had with regard to the language used in the questions.

RESULTS AND DISCUSSION

Overview

The results from the complete sample (n=545) varied considerably from question to question. The great majority of students were able to identify the correct expressions in Questions 7a and 8a which demonstrated the commutative property (93.8%) and inverse relationship (86.6%) respectively. However, a very small number of students could adequately explain why the commutative property (1.8%) or the inverse relationship (1.8%) work. The results from the full sample are shown in **Table 2**.

Table 2. Correct responses to the selected questions shown as percentages

Question and target aspect	n=545	%
1c Numbers of equal groups for 8 x 7 or story to reflect that	249	45.7
2b(iii) Draws array to represent 4 x 3	307	56.3
3(iii) Writes commutated factor pairs for 24 and/or 30	206	37.8
4a(iii) Identifies all factors of 24 and/or 30	179	32.8
7a Identifies expressions showing the commutative property	511	93.8
7b Explains commutative property appropriately	10	1.8
8a Identifies expressions showing inverse relationship	472	86.6
8b Explains inverse relationship appropriately	12	2.2

Table 3. Percentages of students by age/year groups recording correct responses to the eight questions

Questions	Year 4 9 yrs	Year 5 10 yrs	Year 6 11 yrs
1c Numbers of equal groups for 8 x 7 or story to reflect	35.3	49.0	52.0
2b(iii) Draws array to represent 4 x 3	62.9	60.8	44.5
3(iii) Writes commutated factor pairs for 24 and/or 30	24.0	46.6	42.2
4a(iii) Identifies all factors of 24 and/or 30	15.0	33.3	49.7
7a Identifies expressions showing commutative property	94.0	92.6	94.8
7b Explains commutative property appropriately	0.6	1.5	3.5
8a Identifies expressions showing inverse relationship	80.8	86.8	87.2
8b Explains inverse relationship appropriately	0.6	2.0	4.0

Table 4. Numbers of students recording various correct responses to the six questions

Number of correct responses for six questions	Number of students (n=545)	%
8 correct responses	3	0.6
7 correct responses	4	0.7
6 correct responses	57	10.5
5 correct responses	95	17.4
4 correct responses	113	20.7
3 correct responses	127	23.3
2 correct responses	102	18.7
1 correct responses	34	6.2
0 correct responses	10	1.8

Just as there were obvious differences in the percentages of correct responses across the different questions, there were also clear differences in the students' responses to particular questions. That is, of the students who identified that the 8 and 7 in the given number fact showed the number of groups and number in each group, not all of them also wrote a word story that reflected that, and vice versa. Similarly, not all students who wrote all factors and showed commutated pairs for 24, also did so for 30. Over the whole sample, there were only 10 students (1.8 %) who did not provide a single correct response to any question. Conversely, there were only seven students, about 1.3%, who correctly responded to seven or eight questions. Hence, it is obvious that the vast majority of students (98.7%) responded correctly to varying numbers of questions. It is also of interest to consider the breakdown of the correct responses in terms of year/age groups. This is shown in **Table 3**.

There is some difference in the level of correct responses for students of different ages. There is almost three times as many students in the 11 year (Year Six) group being able to identify all factors of 24 or 30 compared to the 9 years (Year Four) group. Similarly, there are more of the older students writing factors as commutated pairs and identifying the numbers of equal groups in a number fact.

It is not clear why this is so and seeking an explanation is not within the scope of this article. However, the observation that less students in the 11 years group (Year Six) depicted the number fact with an array is of interest and will be discussed in the context of teacher comments in a later section.

The main focus here was whether or not students were displaying a conceptual understanding of the properties and relationships and were able to connect the ideas shown in the definition provided earlier. For example, rather than simply being able to identify properties and relationships, it was of interest to see if students could explain why they worked. The number of students providing various numbers of correct responses (from 0 to 8) is shown in **Table 4**.

Some observations can be made from the data generated from the eight questions. First, analysis of the responses to Questions 1c (equal groups), 2b (iii) (arrays), 3(iii) (writing factors in pairs), and 4(iii) (identifying factors), which deal with the conceptual tools for explaining why the inverse relationship and commutative property work, indicates that 10.3% of the students (n=56) correctly responded to all four questions. However, only three were able to adequately explain both the relationships. Second, within the 69.0% of students who provided one, two, three, or four correct responses, there was no particular pattern or combination of correct answers. That is, some students identified numbers of equal groups, drew an array, and identified factors but did not write factors as commutated pairs, while other students wrote factors in commutated pairs for one number, and drew an array but did not identify numbers of equal groups or factors of the other given number. In summary, apart from the three students who did provide correct responses for all questions, students did not connect their knowledge of the first four questions to the commutative property or inverse relationship.

Questions 1(b) and 1(c) Numbers of Equal Groups

Altogether, 249 students (45.6%) were able to articulate that the numbers in a basic multiplication fact (8 x 7) represented the number of groups and the number in each group, and/or wrote an appropriate word problem which reflected that. Only 37 students were able to discuss in terms of numbers of equal groups in Question 1(b) but 155 students were able to write a word problem or story problem that reflected an understanding of equal groups as in Question 1(c). Also, 94 students wrote correct responses for both questions. An interesting observation is that eight students, all from the same class, described the number fact (8 x 7) in terms of factors. For example, Student RDA (Class HP5) said "The 8 and 7 are factors so you times them to make a product". No students from any other classes described those numbers – in Question 1(b) – as 'factors'. However, of the eight students who did so, only one was also able to provide all the factors of 24 or 30, and show factors as commutated pairs. This suggests that there may have been some explicit teaching about factors to that class but the students had not connected their understanding to different situations or contexts. The word problems or story problems written were typified by Student DRA (Class HP5) who said "There are seven sweets in each jar and there are eight jars. How many sweets altogether?" Student BHA (Class HP5) wrote a similar story: "There are eight boxes and there were seven apples in each box. How many apples were there altogether?"

Questions 2(a) and 2(b) Arrays

A total of 307 students (53.4%) represented the number fact 4 x 3 as an array, and of them 83 responded to Question 2(b) by showing a rotated or commutated array. Despite holding knowledge that the array could be rotated, only ten of the 83 students were able to explain how the commutative property works, two of whom drew two commutated arrays to demonstrate it. Further to this, only two students explained the commutative property by drawing two commutated arrays – Student ALE (Class PA5) and Student SHE (Class RA6).

Question 3(iii) Commutated Factor Pairs

Altogether, 269 students (46.8%) wrote at least one pair of factors, as for example, (8, 3), (8 x 3), or (8 and 3). This may suggest the existence of a level of understanding about the multiplicative situation in that if one factor and a product (in this case 24) is known, then the other factor is known. As well, 206 of those students went on to show at least three sets of commutated pairs of factors for either 24 or 30. These were generally represented as 6 x 5 and 5 x 6, 10 x 3 and 3 x 10, though some students such as DAT (Class HP6) wrote them vertically as 1 x 24 = 24, 2 x 12 = 24, 3 x 8 = 24, and so on to 12 x 2 = 24 and 24 x 1 = 24. Notwithstanding, only ten students were able to connect their knowledge of factor pairs and use it to explain the commutative property or the inverse relationship. Some examples of strong explanations of those properties follows shortly.

Question 4 Writing Factors of 30 (or 24)

The majority of students (456 or 79.3%) wrote at least one factor for 24 and/or 30. This seems to indicate a knowledge of what a factor is, even though the students may have been unable to articulate that. A total of 179 students (31.4%) were able to show all of the factors of 24 and/or 30. These were recorded in a variety of ways with no representation being more common than any other. Some students listed the factors in pairs (e.g., 1 x 24, 2 x 12, 3 x 8 and 4 x 6) and others listed them in order (e.g., 1, 2, 3, 4, 6, 8, 12, and 24). Again, although a large number of students were able to show at least some factors and almost a third of the cohort could list all the factors of a number, only a small number could connect that knowledge to an articulation of how the inverse relationship worked.

Question 7(b) Commutative Property.

The title of this article mentions the 'capacity' of children to think multiplicatively. The responses of some students to Question 7(b) suggest that such a capacity exists. Ten students were able to explain how the commutative property works and it has been noted already that Student ALE (Class PA5) and Student SHE (Class RA6 drew two commutated arrays (a 6 x 17 array and a 17 x 6 array) to explain commutativity. Two other students made mention of factors in their responses. Student ZIX (Class PA5) said: 'You'll get the same answer as when you switch the factors around. It is also the same with addition' and Student NAY (Class PA5) said 'If the factors are the same, even if they are switched around, the product will always be the same'. It is of interest that both students are from the same class, as were the eight students from another class who used the term 'factor' to describe the situation with the number fact in Question 1(b). Student BRO (Class PH6) said something similar but did not use the term 'factor': '17 x 6 will give the same answer [as 6 x 17] because it's the same numbers and in multiplication, it doesn't matter which order you do it in'. Five other students justified that 17 x 6 would give the same answer as 6 x 17 by saying 'Because 6 groups of 17 will give the same answer as 17 groups of 6' (or similar wording).

A further 13 students in two classes – PH5 and PH6 – simply responded with 'Because it's the commutated law [or commutative law]'. These responses have been discounted as explanations because they don't actually say how or why the property works. It is worth noting that, out of the 545 students from 19 classes, these 13 students (from two classes at the same school) were the only students to use the words 'commutative' or 'commutated'. This strongly suggests that, although there seems to have been some explicit teaching about the commutative property in those two classes, students have not connected their knowledge of factors to explaining why the property works.

Question 8(b) Inverse Relationship

As with the previous question, it is suggested that responses from some students to Question 8(b) also indicate that the 'capacity' to think multiplicatively exists. Five of the ten students who could articulate why the inverse relationship worked did so in terms of factors. Student RCO (Class PH5) said '144 ÷ 6 will help you because you can find the other factor. It is also the same for 144 ÷ 24' and JOS (Class SA6B) said 'Because 24 is the factor you used to get 144 so that's the answer if you divide it by 6, the other factor'. Student JES (Class PA6) explained it this way: 'Yes, in the first sum, (24 x 6 = 144), it gives you the factors so when you divide, you know that 6 lots of 24 go into 144 (do a swap around) and you have the answer'. Student RDA (Class PH5) gave a similar answer while Student NAY (Class PA5) said that 'Because from the two problems, the product divided by the factor will give the missing factor'.

Five other students explained the relationship in terms of equal groups. Student ZIX (Class PA6) said 'It is correct because if 24 groups of 6 marbles equals 144 and you grab all the 144 marbles and divide them into 6 equal groups, you'll get 24 marbles in each group'. Student BEN (Class PA4), RAE (Class PA6) and Student ALE (Class RA6) also described their thinking in terms of equal groups. Student AIS (Class PA5) offered a different explanation, saying 'Yes, because multipliers can also be used to find quotients, e.g., 10 ÷ 2 = 5, as 5 x 2 = 10'. This demonstrates some very strong understanding of the multiplicative situation and invites the question, 'how did this student come to understand this point?' Whatever the case, it is certainly indicative of the capacity to think multiplicatively. A further five students tried to explain the reason as division being the 'inverse operation' or that 'division is pretty much the opposite of multiplication'. Whilst division is indeed the inverse of multiplication, neither explanation demonstrates an understanding of why the relationship works, in terms of factors and/or equal groups.

Teacher Responses

It can be seen from the above results and associated discussion that most students in the sample (n=545) knew about some of the content and ideas that underpin a conceptual understanding of the multiplicative situation but that they overwhelmingly failed to connect their knowledge to other ideas. Now it is important to consider what level of understanding is displayed by some of the teachers whose students were part of the sample. Teachers at School (PH) were given work samples generated from the MTQ and asked to comment on them, and they were also interviewed about aspects of multiplicative thinking.

They were shown a sample using arrays and asked what the sample showed about the students' understanding of the mathematics involved and also what teaching strategies they would employ to help each student. The samples are shown in **Figure 2**.

Tommy and Jamie were asked to represent 3 x 4 with tiles. They responded in the following ways, respectively.

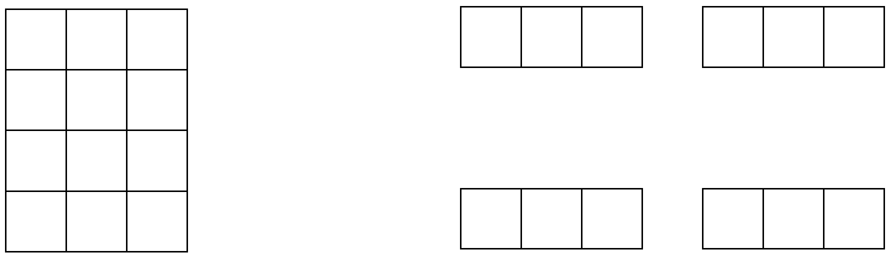

Figure 2. Work samples from Tommy and Jamie from teacher response sheet

Teacher A

Teacher A responded with the following comment: 'Both have used arrays. Tommy perhaps sees the commutative aspect better. Jamie perhaps is comfortable with repeated addition'. No teaching strategy was suggested. When interviewed at a later date, Teacher A described how his students responded to a task based on the following combination problem: 'If I have seven shirts and three pairs of shorts, how many different combinations of shirts and shorts can I wear?' Teacher A was surprised when a number of his students drew an array to work it out, in preference to the method they had generally used before (i.e., a tree diagram). He had not suggested the use of the array. He was then asked specifically about arrays.

INT: How much do you use the array in your general teaching?

TEACHER A: If I'm honest with you, hardly ever . . . hardly ever, and it's interesting that they all picked up on it [using the array]. Perhaps it's something I should be bringing into my practice.

Teacher A also said that as students move into the upper primary years, they are reluctant to use tools like the array and the number line and view them as things that are used in the early years.

There are several points worthy of note about the responses of Teacher A.

- He stated that both students in the sample used an array where they clearly did not. The second student depicted the number fact in terms of equal groups.
- Teacher A did not identify that in both cases the students' drawings showed 4 x 3 and not 3 x 4 as in the question.
- Even though he does not use the array and suggested that students are reluctant to do so, it was the array that was the preferred method of representation for the combination problem task he described.
- His comment that Tommy 'perhaps sees the commutative aspect better' is interesting and raises a couple of questions – Has Teacher A seen Tommy's array as being rotated from a 4 x 3 array? Does the fact that he does not use the array provide a reason why seven students in his class knew about the 'commutated law' or 'commutative law' but could not explain why it works in terms of an array?

Teacher B

Teacher B responded to the work samples shown in **Figure 2** by saying 'Both have responded with logical answers, Tommy 3 by 4 and Jamie 4 lots of 3. Jamie may have some misconception'. It is of interest that Teacher B did not identify that Tommy used an array and her response about 'logical answers' is quite vague. Again, as with Teacher A, no suggestion about strategies has been made. Teacher B was also asked specifically about the use of the array.

INT: Do you use the array a lot?

TEACHER B: I think lower down the school they use arrays more but not after Key Stage One [approximately 7 years old]".

Teacher B also noted that the use of manipulatives falls away quickly once students move out of the early primary years.

Teacher C

Teacher C responded to the work samples shown in **Figure 2** by saying 'Tommy and Jamie both understand that the multiplication requires either 3 lots of 4 or 4 lots of 3'. As a strategy, he suggested 'Work on representing calculations in different ways to deepen understanding'. Teacher C did not identify that Tommy had used an array and his explanation of the students' responses is unclear. Also, he did not identify that both students had depicted a 4 x 3 situation whereas the question was based on 3 x 4. Teacher C's response to the interview question about the array suggested that he may not have a clear idea of what an array actually is.

INT: In Year 6, do you use the array in your teaching?

TEACHER C: Is that the . . . [pause] . . .

INT: Rows and columns of dots.

TEACHER C: [further pause] . . . No we don't, not in Year 6. No.

The collective responses from Teachers A, B, and C suggest that the array is not used in the upper primary classes in that school. This may serve to indicate why students in the school sample could not articulate why the commutative property works. It also provides a possible explanation, if this situation exists in general across the complete sample, as to why the depiction of the 4 x 3 number with an array declined with older students. That is, **Table 3** shows that in the Year 4 (9 years) cohort, 62.9% used an array, in the Year 5 (10 years) cohort, 60.8% used an array, and in the Year 6 cohort, 44.5% did so.

Teachers D, E, and F

Further data were collected from interviews with three teachers at another school. These teachers were keen to follow up the use of the Multiplicative Thinking Quiz (MTQ) and were provided with a number of tasks which highlighted aspects of multiplicative thinking identified in the MTQ results. The responses of Teachers D, E, and

F reflect some important learning in terms of their mathematical content knowledge and they also highlight how that knowledge was previously lacking. When asked about their level of awareness, Teacher D said,

> "I'd never heard of things like the associative property [sic]. I understood that 5 x 7 is the same as 7 x 5 but I'd never heard those concepts. It's hard to explicitly teach kids that, if you're not naming them and understanding them".

Given the evidence about student understanding already presented, this is a particularly powerful comment and serves to underline a likely reason for students not being able to fully understand and connect mathematical ideas. Teacher E responded,

> "With arrays, the bit for me that really helped was seeing an example of that strategy being used. . . I'd never thought of teaching it that way nor did I understand the concepts beneath that".

Teacher F said,

> "Once I started using arrays, I could see where that fits into what they should know, so now I do arrays, knowing that the length is one factor and the width is one factor . . . the kids have a strategy for understanding what factors actually are".

These responses are powerful and indicate two things – that the teachers' initial understanding was lacking and more importantly, that it is eminently possible to develop teacher knowledge when the teachers are receptive.

Teachers D, E, and F were also asked about how they would have reacted to student responses to questions such as those on the MTQ before they had become more aware of the underpinning mathematics. Teacher E said that they would 'randomly plugging holes' and Teacher D added,

> "We wouldn't have been plugging those gaps in the past because we wouldn't have known that they existed . . . if we'd asked them what an array was, they wouldn't have known but we wouldn't have recognised that as a gap".

Teacher F's comment about curriculum documents highlights another issue about teacher knowledge, that is, that teachers need to interpret curricula to successfully teach the content contained in it, but to do that, they need a rich understanding of the key concepts. Teacher F said,

> "If you look at the curriculum documents, it's [multiplicative thinking] not stated explicitly anywhere, it's probably inferred that you would do it, but if you don't have any knowledge of it . . . we had no knowledge of it originally, we weren't doing it and I don't think the curriculum is explicit enough".

Again, the comments from these teachers are powerful in that they highlight the impact of inadequate teacher knowledge on the practice of teachers. In particular, if teachers are unaware of what students do not know because they themselves do not know the content and understand the connections, the shortcomings in student knowledge will continue to go undetected. In addition, the curriculum (in this case the Australian Curriculum: Mathematics) may be seen by the teachers as lacking in advice for teachers about how to teach aspects of multiplicative thinking in a connected way.

The final word on teacher knowledge and practices comes from a teacher at another project school. She had observed students in her class doing the MTQ under the supervision of one of the research team and had noticed that many students had difficulty in answering the questions about factors. Following the administration of the MTQ, she remarked to the researcher, "I don't understand it. They struggled with the factor questions but I taught factors about three weeks ago". The significance of this comment relates to what Askew (2016) said about mathematical ideas being 'models for' rather than 'models of'. He was talking about arrays but the same can be applied to factors which need to be seen not as a 'model of' a number, but rather as a 'model for' understanding relationships between numbers, and then as 'tools for thinking with'. Like arrays, factors are not something to be 'taught' and then left but rather they need to be embedded in the daily discourse of the mathematics classroom so that students can see how they relate to, and can be used to explain other ideas.

CONCLUSIONS AND IMPLICATIONS

The research questions posed were as follows:

- To what extent do children in Years 4, 5 and 6 (i.e., of ages 9, 10, and 11 years) think in a connected way to explain multiplicative concepts?
- What is the extent of teacher knowledge of multiplicative concepts?

It is apparent from the evidence presented that primary school students have the capacity to think multiplicatively and in a connected way but that capacity is often not fully developed. Most students have knowledge of aspects of multiplicative thinking as defined in the conceptual framework (**Figure 1**) but often do not connect that knowledge to enable them to understand multiplicative concepts. There is also evidence to suggest that teacher content knowledge may be unconnected and though the teacher data presented here is derived from a relatively small sample, there is clearly evidence of unconnected knowledge of multiplicative concepts in both students and teachers. It certainly seems to be an issue that warrants further research. Some conclusions worthy of consideration can be drawn from the presented data and they are listed here.

- The majority of students know about at least some aspects of multiplicative thinking.
- Students know about different aspects of the mathematics that underpins multiplicative thinking and not all students hold the same mathematical knowledge as do other students.
- A small minority of students in each of the three year/age levels were able to connect their knowledge of some of the mathematics and use it to explain other mathematical ideas in a conceptual way.
- Some students held a knowledge of all contributing mathematical ideas but did not connect their knowledge to understand multiplicative ideas in a conceptual way.
- The array is a central mathematical idea that underpins the multiplicative situation.
- Teacher content knowledge, specifically in relation to arrays, may be limited, at least within the places from where the sample for this study originated.
- Arrays are not widely used in middle and upper primary classes.
- Incomplete content knowledge of teachers is likely to mean that student misconceptions and misunderstandings may not be identified.
- Teaching of mathematical ideas like arrays, factors, and equal groups needs to be done in a sustained way where those ideas are more than 'models of' mathematical ideas but rather 'tools for working with' and understanding those ideas.

Numerous people have lamented the lack of understanding that primary school students have about mathematical structure (Anthony and Walshaw, 2002; Warren and English, 2000). Feldman (2014) has indicated that a strong understanding of structure (such as factorisation) prepares students to deal with more complex ideas in flexible and efficient ways. Yet, the evidence presented here suggests that students hold 'pieces of knowledge' rather than the complete connected picture. The conceptual framework for this study is an attempt to represent some of the structure of multiplicative concepts. It is clear that many students hold knowledge of components of that structure but have not developed an understanding of how those components are connected, nor are they able to articulate such understanding through the use of essential mathematical language.

Thanheiser et al. (2013) suggest that the procedural view of mathematics held by teachers and PSTs is not helpful in preparing primary school students to understand mathematical structure. The findings here certainly do not suggest that teaching to develop connected understanding of concepts is prevalent in primary school classrooms. For example, out of a sample of 545 students, 511 (93.9%) can identify the commutative property but most of them explain it in terms of a 'switch around' or a 'turn around', something that they have probably been taught and assimilated as a procedure. Similarly, 472 students (86.8%) can identify the inverse relationship between multiplication and division but the vast majority explain it by saying that 'multiplication and division are from the same family' or something similar. Again, this displays a familiarity with the procedure but not necessarily the concept. As well, the words 'commutated' or 'commutative' were only used by 13 students from two classes, yet none of them said anything other than "It's the commutated law". They were not able to explain why it worked, suggesting that they were taught the name but not the understanding – a procedural approach. The importance of teachers holding a connected view of mathematical ideas was mentioned earlier in relation to Ma's (2010) knowledge packages. It is also evident in the 'connection' element of the Knowledge Quartet (Rowland, Huckstep and Thwaites, 2005) which enables teachers to structure lessons and sequences of lessons in a coherent way. Similarly, Ball, Thames and Bass (2008) conceptualised 'horizon content knowledge', part of which is teachers' ability to connect the specific mathematics at hand to other and larger mathematical structures and principles.

The conceptual framework presented in **Figure 1** is the basis of this article, suggesting that the mathematical structure could be considered to contain those links and connections between the array, factorisation, factor pairs, and equal groups, and the two properties or relationships – commutativity and inverseness. It has been shown that most students know about some of the four underpinning ideas but have not connected them to articulating an understanding of the properties. Only four students out of 545 were able to make those connections. Does this mean that they did so as a result of explicit teaching or did the 'join up the dots' for themselves? The evidence presented here suggests the latter. The implications for teacher educators, education systems, and professional learning providers, are clear – unless PSTs and practising teachers have a rich understanding of mathematical structure in a conceptual rather than procedural way, they will not be equipped to make connections between mathematical ideas explicit for their students.

REFERENCES

Anthony, G. and Walshaw, M. (2002). Swaps and switches: Students' understandings of commutativity. In B. Barton, K. C. Irwin, M. Pfannkuch and M. O. J. Thomas (Eds.). *Mathematics Education in the South Pacific* (Proceedings of the 25th annual conference of the Mathematics Education Research Group of Australasia, Auckland, pp. 91-99). Sydney: MERGA.

Askew, M. (2016). *Transforming primary mathematics: Understanding classroom tasks, tools and talk*. Oxon: Routledge.

Askew, M., Brown, M., Rhodes, V., Wiliam, D. and Johnson, D. (1997). *Effective teachers of numeracy: Report of a study carried out for the Teacher Training Agency*. London: King's College, University of London.

Ball, D. L., Thames, M. and Bass, H. (2008). Content knowledge for teaching: What makes it special? *Journal of Teacher Education*, 59(5), 389-407. https://doi.org/10.1177/0022487108324554

Baroody, A. J. (1999). The roles of estimation and the commutativity principle in the development of third graders' mental multiplication. *Journal of experimental Psychology*, 74, 157-193. https://doi.org/10.1006/jecp.1999.2524

Charles, R. I. (2005). Big ideas and understandings as the foundation for early and middle school mathematics. *NCSM Journal of Educational Leadership*, 8(1), 9-24.

Chick, H., Pham, T. and Baker, M. (2006) Probing teachers' pedagogical content knowledge: Lessons from the case of the subtraction algorithm. In P. Grootenboer, R. Zevenbergen and M. Chinnappan (Eds.), *Identities, cultures, and learning spaces* (Proceedings of the 29th annual conference of the Mathematics Education Research Group of Australasia, Canberra, pp. 139-146). Adelaide: MERGA.

Clarke, D. M., Clarke, D. J. and Sullivan, P. (2012). Important ideas in mathematics: What are they and where do you get them? *Australian Primary Mathematics Classroom*, 17(3), 13-18.

Davis, B. (2008). Is 1 a prime number? Developing teacher knowledge through concept study. *Mathematics Teaching in the Middle School*, 14(2), 86-91.

Downton, A. (2008). Links between children's understanding of multiplication and division and solution strategies for division. In M. Goos, R. Brown and K. Makar (Eds.), *Navigating currents and charting directions* (Proceedings of the 31st annual conference of the Mathematics Education Research Group of Australasia, pp. 171-178). Brisbane: MERGA.

Downton, A. (2013). Making connections between multiplication and division. In V. Steinle, L. Ball and C. Bardini, (Eds.), *Mathematics education: Yesterday, today, and tomorrow* (Proceedings of the 36th annual conference of the Mathematics Education Research Group of Australasia, pp. 242-249). Melbourne, VIC: MERGA.

Feldman, Z. (2014). Rethinking factors. *Mathematics Teaching in the Middle School*, 20(4), 231-236. https://doi.org/10.5951/mathteacmiddscho.20.4.0230

Gojak, L. M. (2013). *It's elementary! Rethinking the role of the elementary classroom teacher*. Available at: http://www.nctm.org/News-and-Calendar/Messages-from-the-President/Archive/Linda-M_-Gojak/It_s-Elementary!-Rethinking-the-Role-of-the-Elementary-Classroom-Teacher/

Hurst, C. (2015). The multiplicative situation. *Australian Primary Mathematics Classroom*, 20(3), 10-16.

Jacob, L. and Mulligan, J. (2014). Using arrays to build multiplicative thinking in the early years. *Australian Primary Mathematics Classroom*, 19(1), 35-40.

Larsen, S. (2010). Struggling to disentangle the associative and commutative properties. *For the Learning of Mathematics*, 30, 37-42.

Ma, L. (2010). *Knowing and teaching elementary mathematics: Teachers' understanding of fundamental mathematics in China and the United States*. Mahwah, NJ: Erlbaum.

Matney, G. T. and Daugherty, B. N. (2013). Seeing spots and developing multiplicative sense making. *Mathematics Teaching in the Middle School*, 19(3), 148-155. https://doi.org/10.5951/mathteacmiddscho.19.3.0148

Mills, J. (2015). Professional knowledge required when teaching mathematics for numeracy in the multiplicative domain. In M. Marshman, V. Geiger and A. Bennison (Eds.), *Mathematics education in the margins* (Proceedings of the 38th annual conference of the Mathematics Education Research Group of Australasia), pp. 429–436. Sunshine Coast: MERGA.

Rowland, T., Huckstep, P. and Thwaites, A. (2005). Elementary teachers' mathematics subject knowledge: The Knowledge Quartet and the case of Naomi. *Journal of Mathematics Teacher Education*, 8(3), 255-281. https://doi.org/10.1007/s10857-005-0853-5

Siemon, D. (2013). Launching mathematical futures: The key role of multiplicative thinking. In S. Herbert, J. Tilley and T. Spencer (Eds.), *Mathematics: Launching Futures* (Proceedings of the 24th Biennial Conference of the Australian Association of Mathematics Teachers, pp. 36-52), Adelaide: AAMT.

Siemon, D., Breed, M., Dole, S., Izard, J. and Virgona, J. (2006). *Scaffolding Numeracy in the Middle Years – Project Findings, Materials, and Resources*, Final Report submitted to Victorian Department of Education and Training and the Tasmanian Department of Education. Available at: http://www.eduweb.vic.gov.au/edulibrary/public/teachlearn/student/snmy.ppt

Siemon, D., Bleckly, J. and Neal, D. (2012). Working with the Big Ideas in Number and the Australian Curriculum: Mathematics. In B. Atweh, M. Goos, R. Jorgensen and D. Siemon, (Eds.), *Engaging the Australian National Curriculum: Mathematics – Perspectives from the Field.* Online Publication: Mathematics Education Research Group of Australasia pp. 19-45.

Skemp, R. (1976). Relational understanding and instrumental understanding. *Mathematics Teaching*, 77, 20-26.

Sullivan, P. (2008). Mathematical Discipline Knowledge Requirements for Prospective Primary Teachers, and the Structure and Teaching Approaches of Programs Designed to Develop That Knowledge. In P. Sullivan and T. Wood (Eds.) *Knowledge and Beliefs in Mathematics Teaching and Teaching Development.* Vol. 1, (pp. 1-9). Rotterdam: Sense Publishers.

Squire, S., Davies, C. and Bryant, P. (2004). Does the cue help? Children's understanding of multiplicative concepts in different problem contexts. *British Journal of Educational Psychology*, 74, 515-532. https://doi.org/10.1348/0007099042376364

Thanheiser, E., Philipp, R. A., Fasteen, J., Strand, K. and Mills, B. (2013). Preservice teacher interviews: A tool for motivating mathematics learning. *Mathematics Teacher Educator*, 1(2), 137-147. https://doi.org/10.5951/mathteaceduc.1.2.0137

Warren, E. and English, L. (2000). Primary school children's knowledge of arithmetic structure. In J. Bana and A. Chapman (Eds.) *Mathematics Education beyond 2000* (Proceedings of 23rd annual conference of the Mathematics Education Research Group of Australasia, Fremantle, pp. 624-631). Sydney: MERGA.

Young-Loveridge, J. (2005) Fostering multiplicative thinking using array-based materials. *Australian Mathematics Teacher*, 61(3), 34-40.

Young-Loveridge, J. and Mills, J. (2009). Teaching Multi-digit Multiplication using Array-based Materials. In R. Hunter, B. Bicknell and T. Burgess, (Eds.) *Crossing divides: Proceedings of the 32nd annual conference of the Mathematics Education Research Group of Australasia (Vol. 2).* Palmerston North, NZ: MERGA.

Zazkis, R. and Campbell, S. (1996). Divisibility and multiplicative structure of natural numbers: Pre-service teachers' understanding. *Journal for Research in Mathematics Education*, 27(5), 540-563. https://doi.org/10.2307/749847

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